Deceiving Cyber Adversaries: A Game Theoretic Approach

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ABSTRACT

An important way cyber adversaries find vulnerabilities in modern networks is through reconnaissance, in which they attempt to identify configuration specifics of network hosts. To increase uncertainty of adversarial reconnaissance, the network administrator (henceforth, defender) can introduce deception into responses to network scans, such as obscuring certain system characteristics.

We introduce a novel game theoretic model of deceptive interactions of this kind between a defender and a cyber attacker, which we call the Cyber Deception Game. We consider both a powerful (rational) attacker, who is aware of the defender's exact deception strategy, and a naive attacker who is not. We show that computing the optimal deception strategy is NP-hard for both types of attackers. For the case with a powerful attacker, we provide a mixed-integer linear program solution as well as a fast and effective greedy algorithm. Similarly, we provide complexity results and propose exact and heuristic approaches when the attacker is naive. Our extensive experimental analysis demonstrates the effectiveness of our approaches.

KEYWORDS

Game Theory; Cyber Security; Security Games

1 INTRODUCTION

Network security is an important problem faced by organizations who operate enterprise networks housing sensitive information and complete important functions. This challenge is highlighted by several recent major attacks which have caused severe damage, such as the Equifax breach in 2017 and Yahoo in 2016 [13, 14]. Criminals who target networks first map it out by using network scanning tools. These tools answer important questions such as: which computers are connected to each other and their IP addresses? What operating system is a computer running? What ports are open and what services are they running? What are the names of associated subnetworks and users? Given answers to all of these questions, an attacker is able to maximize his chance of successfully infiltrating the network and gaining a foothold. To gain such information, attackers can use a suite of requests using tools such as NMap [19].

To protect against attacks, network administrators use techniques such as the whitelisting of applications, locking down permissions, and immediately patching vulnerabilities [16]. An interesting direction of research is the use of deception as a framework to improve cybersecurity defenses [4]. [1] explores ways to achieve deception through OS and service obfuscation to thwart potential attackers. Instead of directly stopping an attack, deceptive techniques concentrate on diverting an adversary to attack non-critical systems or honeypots using deceptive views of the network state. Essentially, approaches for deception focus on making it difficult for an attacker to accurately identify information about systems on the network using tools like NMap. However, one drawback of most of these previous approaches is that they do not adequately model the adversarial nature of the cybersecurity domain.

Experienced attackers attempting to infiltrate a network spend a significant amount of time during the reconnaissance phase of their attack to find vulnerabilities throughout the network by mapping out the network through NMap scans, stealth SYN scans, TCP connections scans along with others [16, 20]. After gathering all of this information, the attacker then mounts their attack on a network. In the cyber domain, the network administrator has an asymmetric information advantage as she knows the true state of the network, i.e., properties of the system such as its hardware type or the operating system, and further, she can control the responses to scans sent by an adversary [2, 8]. By hiding or lying about part of each system's configuration, the defender could make it significantly harder for the adversary to determine the true vulnerabilities present in systems on the network. Since exploits generally rely on specific vulnerabilities and versions of software, incorrectly identifying a system's software information decreases the likelihood of a successful attack.

Our work concentrates on how the defender can benefit the most from determining a mix of true, false and obscure responses to deceive the attackers. To highlight the defender's advantage, consider a network with 1 system running NGINX and 2 running Tomcat. Suppose the adversary has a specific exploit for NGINX. The adversary scans all systems to find the one running NGINX and then deploys his exploit. However, if the defender can lie about

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the webserver, the adversary potentially has to test his exploit on all systems to infiltrate the network. This process increases the time spent by the adversary to infiltrate the network (which gives the defender time to mount a better defense) and increases the chances the defender catches an attack. The problem for the defender then is to determine how to alter the adversary's perception of the network to minimize her expected loss from an attack.

Our first contribution is the Cyber Deception Game (CDG) model which captures the strategic interaction between the defender and an adversary in network security. In this game, the defender chooses how systems respond to scans and the attacker chooses which system to attack based on the responses. For our second contribution we show that finding the defender's optimal strategy against a powerful attacker who knows the defender's exact deception scheme in CDGs is NP-hard and provide a Mixed Integer Linear Program (MILP) to compute the optimal response scheme. We then propose a greedy algorithm to quickly find good defender strategies in CDGs which is shown to perform well experimentally in a fraction of the time of the MILP. Third, we show that surprisingly the problem is still NP-hard when faced with naive attackers who act according to prior fixed utilities given budget constraints, and propose an algorithm to provide the exact solution. Finally, we present experimental results showing the scalability of our solution techniques and a comparison of the solution quality of proposed techniques for both types of adversaries.

2 RELATED WORK

The use of game theory for security has been studied extensively, which we discuss in Section 3. Game theory has also been studied in the context cybersecurity problems [5, 18, 24, 25]. [11, 12, 17, 23] study a honeypot selection game where a defender chooses the properties of the network where the attacker can use probe actions to test the network and his actions are represented as attack graphs. [10] studies a signaling game where the defender signals to an adversary if a system is either real or a honeypot when the adversary performs a scan. [22] extends the signaling game to account for an adversary who can gain evidence about the true state of a system. In our work, we consider a game scenario in which the defender determines the optimal way to respond to scans sent by a potential adversary given a set of possible responses. Further, we explore different types of adversaries with varying awareness of deception.

Deception has also been widely studied as a means to improve the protection of enterprise networks from potential hackers and intruders [1, 3]. [2] uses a graph theoretic approach to confuse a potential attacker by manipulating his view of systems on the network. However, this work focuses on finding a view which is measurably different from the true state and does not adequately model the response of a strategic adversary. [15] is the most closely related to our work. The authors study how to respond to an attacker's scan queries using an annotated probabilistic logic model. We provide a complimentary view using game theory to determine how a defender manipulates scan responses to confuse an attacker's view of systems on the network. We also study varying adversary models, which can have significant impact on the defender's optimal strategy which is not explored in [15].

3 CYBER DECEPTION GAME

The *Cyber Deception Game* (CDG) is a zero-sum Stackelberg game between the defender (e.g., network administrator) and an adversary (e.g., hacker). The defender moves first and chooses how the systems should respond to scan queries from an adversary, and the adversary subsequently moves by choosing a system to attack based on the responses. Despite the similarities with game-theoretic models in security domains, such as [6, 7, 26], there are two key differences. First, the defender can only commit to a pure strategy and not an arbitrary mixed strategy. This is because, in these domains, network administrators modify the network very infrequently, and thus, the attackers' view of the network is static. Second, there are no explicit security resources for the defender in CDGs. Consequently, the existing approaches for solving standard Stackelberg games in security domains, cannot be directly applied. The various components of the game and the aforementioned model characteristics are described in detail as follows:

Systems and True Configurations. The defender aims to protect a set K of systems, from possible exploits and intrusions. Each system has certain attributes, e.g., an operating system, an antivirus protection mechanism, services hosted, etc. These attributes altogether constitute the *true configuration* (TC) of the system. We denote the set of all possible TCs by F. We consider a zero-sum game. Each system has an associated utility, which captures how much the adversary would get by attacking it. This utility solely depends on the TC of the system — each $f \in F$ induces a utility denoted by U_f to any system that is assigned f. U_f can be negative if the security level of the system is so high that the attacker's efforts end in vain or the attacker gets fake data from a seemingly successful attack, leading to a loss in the end. It follows that, the *true state of the network* (TSN) can be represented as a vector $N = (N_f)_{f \in F}$, where $N_f \in \mathbb{Z}_{>0}$ denotes the number of systems on the network which have a TC f and $\sum_{f \in F} N_f = |K|$ (We assume $N_f \neq 0$, since such a TC simply need not be considered).

Observed Configurations. The adversary attempts to gain information about every system on the network, via probes and scans. By scanning a system, the adversary observes certain attributes, which constitute the *observable configuration* (OC) of the system. We denote the set of possible OCs by $\tilde{F}$. We assume that it is possible for the defender to make some of the observable attributes of a system appear different than what they truly are (e.g., altering the TCP/IP stack of a system, spoofing a running service on a port). By means of such alterations at her disposal, the defender controls the OC an attacker sees when probing a system. Note that it may not be possible for an arbitrary TC $f \in F$ to be made to appear as an arbitrary OC $\tilde{f} \in \tilde{F}$ — we call such a constraint a *feasibility* constraint, and these are denoted by a (0,1)-matrix π. Iff $\pi_{f,\tilde{f}} = 1$, we say f can be *covered*, or *masked* with $\tilde{f}$. We denote the set of OCs which can mask a TC f, by $\tilde{F}_f = \{\tilde{f} \in \tilde{F} \mid \pi_{f,\tilde{f}} = 1\}$, and similarly, the set of TCs which can be masked by an OC $\tilde{f}$, by $F_{\tilde{f}} = \{f \in F \mid \pi_{f,\tilde{f}} = 1\}$.

From the adversary's perspective, two systems having the same $\tilde{f}$ as their OC are indistinguishable, and hence, his *observed state of the network* (OSN) can be represented as a vector $\tilde{N} = (\tilde{N}_{\tilde{f}})_{\tilde{f} \in \tilde{F}}$

where $\tilde{N}_{\tilde{f}} \in \mathbb{Z}_{\geq 0}$ denotes the number of systems which have an OC $\tilde{f}$. As is the case with the TSN N, we must have $\sum_{\tilde{f} \in \tilde{F}} \tilde{N}_{\tilde{f}} = |K|$.

We assume that masking a TC f with an OC $\tilde{f}$, has a cost of $c(f, \tilde{f})$ incurred by the defender, which typically captures the monetary costs for deploying network modifications necessary for such a deception.

Defender Strategies. Naturally, F, $\tilde{F}$, π, c and N are known to the defender. Given all this information, the defender must decide her strategy — for each TC f, she must decide how many of the N_f systems having TC f, should be assigned the OC $\tilde{f}$, where $\tilde{f} \in \tilde{F}_f$. Thus, any possible strategy can be represented as a $|F| \times |\tilde{F}|$ matrix ϕ having non-negative integer entries, with $\phi_{f,\tilde{f}}$ representing the number of systems having TC f and OC $\tilde{f}$. Hence, ϕ must satisfy

$$\phi_{f,\tilde{f}} \in \mathbb{Z}_{\geq 0} \quad \forall f \in F, \forall \tilde{f} \in \tilde{F} \tag{1}$$

Since the TSN N is fixed, ϕ must also satisfy

$$\sum_{\tilde{f} \in \tilde{F}} \phi_{f,\tilde{f}} = N_f \forall f \in F \tag{2}$$

Since feasibility constraints π are specified, ϕ must also satisfy

$$\phi_{f,\tilde{f}} \leq \pi_{f,\tilde{f}} N_f, \forall f \in F \, \forall \tilde{f} \in \tilde{F} \tag{3}$$

Finally, since setting any OC on a system has an associated cost, we assume that the defender cannot afford the total cost to exceed a limit B, which we call the budget constraint. Formally, ϕ must also satisfy

$$\sum_{f \in F} \sum_{\tilde{f} \in \tilde{F}} \phi_{f,\tilde{f}} \, c(f, \tilde{f}) \leq B \tag{4}$$

The set of strategies ϕ which satisfy the constraints (1), (2), (3), and (4), is denoted by Φ.[1] When the defender plays $\phi \in \Phi$, the resulting OSN $\tilde{N}$ is given by $\tilde{N}_{\tilde{f}} = \sum_{f \in F} \phi_{f,\tilde{f}} \, \forall \tilde{f} \in \tilde{F}$.

Adversary Strategies. Depending on the defender's strategy, the adversary observes $\tilde{N}$ as described above. Since all the systems having the same OC $\tilde{f}$ are indistinguishable to the adversary, he must be indifferent between all such $\tilde{N}_{\tilde{f}}$ systems when deciding which system to attack. As a result, we assume that he attempts to choose the OC $\tilde{f}$ which gives him the highest expected utility (described momentarily), and attack all the $\tilde{N}_{\tilde{f}}$ systems having this OC with an equal probability. In short, we say "the adversary attacks an OC $\tilde{f}$" to mean he attacks all the systems having OC $\tilde{f}$ with an equal probability. A general mixed strategy for the adversary is to attack the set of OCs with any probability distribution. However, since there always exists a pure best-response strategy in any game, it suffices to consider the adversary's strategies as simply attacking a particular $\tilde{f}$.

[1] The feasibility constraints can simply be captured via the budget constraint by setting the costs of infeasible assignments to be higher than the budget. However, they are essential in the model, since, in some cases, having no budget constraint allows an efficient solution to the problem (e.g. Section 7), while still having the very practical feasibility constraints keeps the problem non-trivial.

Utilities. When the defender plays a strategy ϕ, the adversary's expected utility on attacking an OC $\tilde{f}$ with $\tilde{N}_{\tilde{f}} > 0$, denoted by $\bar{U}_{\tilde{f}}(\phi)$ — or, as $\bar{U}_{\tilde{f}}$ for simplicity, when the underlying ϕ is unambiguously understood — is given by

$$\tilde{U}_{\tilde{f}} = E[U_f | \phi, \tilde{f}] = \sum_{f \in F_{\tilde{f}}} P(f|\phi, \tilde{f}) U_f = \sum_{f \in F} \frac{\phi_{f,\tilde{f}}}{\tilde{N}_{\tilde{f}}} U_f \tag{5}$$

(5) follows from computing $P(f|\phi, \tilde{f})$ using the fact that out of $\tilde{N}_{\tilde{f}}$ systems having an OC $\tilde{f}$, $\phi_{f,\tilde{f}}$ have a TC f. Since the game is zero-sum, the defender's expected utility is $-\tilde{U}_{\tilde{f}}$ when $\tilde{f}$ is attacked. Note the attacker cannot attack an OC $\tilde{f}$ with $\tilde{N}_{\tilde{f}} = 0$, or equivalently, his expected utility is $-\infty$ if he does so.

Next, we illustrate the model using a simple example.

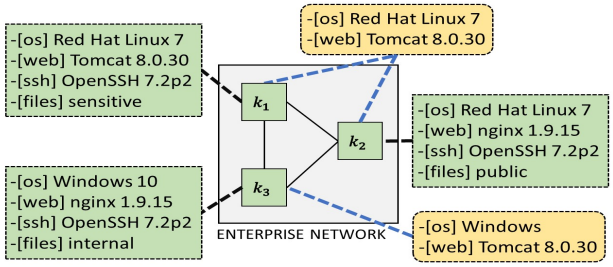


Figure 1: Simple example of an enterprise network.

Figure 1 shows a simple example enterprise network which will be used as a running example. We have a set of systems $K = \{k_1, k_2, k_3\}$, set of TCs $F = \{f_1, f_2, f_3\}$ (shown in Figure 1 as the green boxes) and set of OCs $\tilde{F} = \{\tilde{f}_1, \tilde{f}_2\}$ (shown in Figure 1 as the yellow boxes). Let the feasibility constraints be given by the sets $F_{\tilde{f}_1} = \{f_1, f_2\}$ and $F_{\tilde{f}_2} = \{f_2, f_3\}$. The TCs are as follows:

f_1 = [[os] L, [web] T, [ssh] O, [files] S]

f_2 = [[os] L, [web] N, [ssh] O, [files] P]

f_3 = [[os] W, [web] N, [ssh] O, [files] I]

For the TCs, the utilities are $U_{f_1} = 10$, $U_{f_2} = 0$, and $U_{f_3} = 6$. The OCs are as follows:

$\tilde{f}_1$ = [[os] L, [web] T] $\quad$ $\tilde{f}_2$ = [[os] W, [web] T]

For simplicity, let all the costs $c(f, \tilde{f})$ to be 0, so that there is essentially no budget constraint. Based on the TCs assigned as shown, the state of the network $(N_f)_{f \in F}$ is $(1, 1, 1)$. When the defender assigns OCs as shown in Figure 1, her strategy ϕ is given by

$$\begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array} \begin{array}{c} \begin{array}{cc} \tilde{f}_1 & \tilde{f}_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

The expected utility of the adversary (loss of the defender) when he attacks $\tilde{f}_1$ or $\tilde{f}_2$ is respectively given by $\tilde{U}_{\tilde{f}_1} = (10 + 0)/2 = 5$ and $\tilde{U}_{\tilde{f}_2} = 6/1 = 6$. Thus, attacking $\tilde{f}_2$ leads to highest expected utility for the attacker.

Adversary Knowledge and Utility Estimation. The attacker's awareness of the deception and the understanding of the defender's strategy may vary. Note that if the adversary is always able to find the OC with highest expected utility, it is the worst case scenario for the defender given the game is zero-sum. An attacker who is fully aware of how the defender send the false responses to scan requests (via insider threats, information leakage, etc.) would have such ability. Formally, we define a *powerful* attacker to be one who knows F, $\tilde{F}$, π, U and ϕ and chooses to attack the OC with the (correct) highest expected utility $\tilde{U}_{\tilde{f}}$ computed through Equation 5. If the defender chooses a strategy that minimizes the expected utility of a powerful attacker, she gets a robust strategy as the defender can be assured that no matter the extent of the adversary's knowledge, no strategy he plays can lead to a greater loss for the defender, in alignment with the minimax principle.

However, the attacker may not be so powerful. On the other end of the spectrum, if the attacker is unaware of the defender's precise deception scheme or has a very limited understanding of situation such that he cannot make any meaningful inference, his decision making would be completely dependent on the observed configurations of the systems and some fixed preferences over OCs in terms of the estimated expected utility. Formally, we define a naive attacker to be one who chooses to attack an existing OC $\tilde{f}$ (i.e., one at least one system is configured to have) with the highest $\bar{U}_{\tilde{f}}$ where $\bar{U}_{\tilde{f}}$ is not dependent on the defender's strategy and is known to the defender. This is also equivalent to the case where the attacker just has a fixed preference of the OCs. We analyze CDGs with powerful attackers in Section 4, and CDGs with naive attackers in Section 5.

4 OPTIMAL DEFENDER STRATEGY AGAINST POWERFUL ADVERSARY

In this section, we compute the defender's optimal strategy in a CDG assuming a powerful adversary. The adversary attacks an OC from the set $\text{argmax}_{\tilde{f}\in\tilde{F}} \tilde{U}_{\tilde{f}}$ and gets an expected utility of $\max_{\tilde{f}\in\tilde{F}} \tilde{U}_{\tilde{f}}$, denoted in short as $\tilde{U}^*(\phi)$, which is also the defender's expected loss. Hence, the defender aims to minimize it by choosing her ϕ from the set $\text{argmin}_{\phi\in\Phi} \tilde{U}^*(\phi)$.

4.1 Computational Complexity

We call the problem of finding optimal defender strategy against a powerful adversary in a CDG as *CDG-Robust*.

We first investigate a special case. The following proposition which provides a tight lower bound on $\min_{\phi\in\Phi} \tilde{U}^*(\phi)$.

Proposition 4.1. *The expected loss of the defender when playing her optimum strategy, is no lower than the average utility of the systems, i.e.,*

$$\min_{\phi} \tilde{U}^*(\phi) \geq U^{Ave}(K) = \frac{\sum_{f\in F} N_f U_f}{|K|}$$

Proof Sketch. *Configuring the systems with different OCs effectively partitions the set K into subsets. Since the average utility of all the systems in all these subsets is $U^{Ave}(K)$, there exist at least one subset whose average utility is no less than $U^{Ave}(K)$. Therefore the highest expected utility for the attacker, which is the maximum average utility of all these subsets, is no less than $U^{Ave}(K)$.* [2]

Thus, even when the defender plays her optimal strategy, the attacker's expected utility is at least $U^{\text{Ave}}(K)$. Consequently, if the inequality becomes tight for a strategy ϕ, it must be an optimal strategy. It is easy to see that the bound becomes tight if and only if $\tilde{U}^*(\phi) = \tilde{U}_{\tilde{f}}(\phi)$, $\forall \tilde{f}$. Clearly, this is true if and only if $\bar{U}_{\tilde{f}}$ is the same for each $\tilde{f}$ set on any system, trivially so, if only a single OC is set on all the systems. Thus,

Corollary 4.2. *If it is feasible for the defender to set the same OC on all the systems making them all indistinguishable to the adversary, doing so is an optimal strategy. Formally, if $\exists \tilde{f}^*$ s.t. $\exists \phi^* \in \Phi$ where $\phi^*_{f,\tilde{f}^*} = N_f, \forall f$, then $\phi^* \in \text{argmin}_{\phi\in\Phi} \tilde{U}^*(\phi)$.*

It is possible to efficiently check if such an OC exists, by enumeration. However, it may not exist, and we show that *CDG-Robust* is NP-hard in general.

Proposition 4.3. *CDG-Robust is NP-hard.*

Proof. We prove the result via a reduction from the Partition problem (*PART*) which is known to be NP-complete. Given a multi-set S of n positive integers that sum up to $2r$, *PART* is the decision problem to determine if S can be partitioned into two subsets S_1 and S_2 such that the sum of integers in S_1, and S_2 is r each. It can be reduced to *CDG-Robust* as follows.

Let the input to *PART* be a set of integers $S = \{s_1, \ldots, s_n\}$ whose elements sum to $2r$. To construct a CDG, let the set of TCs be $F = \{f_1, \ldots, f_n\} \cup \{f_{n+1}, f_{n+2}\}$, with utilities $U_{f_i} = s_i$ for each $i \in \{1, \ldots, n\}$ and $U_{f_{n+1}} = U_{f_{n+2}} = -r$. Next, let there be $n+2$ systems, each having a different TC. Let the set of OCs be $\tilde{F} = \{\tilde{f}_1, \tilde{f}_2\}$, with $\tilde{F}_{f_i} = \tilde{F}$ for each $i \in \{1, \ldots, n\}$, and $\tilde{F}_{f_{n+1}} = \{\tilde{f}_1\}$, $\tilde{F}_{f_{n+2}} = \{\tilde{f}_2\}$. Let all the costs be 0 so that the budget constraint can be ignored. Assuming the adversary to be powerful, these components completely define a *CDG-Robust* problem.

Note that, by Proposition 4.1 and the fact that $\sum_f U_f = 0$, we know that the optimal strategy ϕ must have $\tilde{U}^*(\phi) \geq 0$. Now, suppose S can be partitioned in subsets S_1 and S_2 such that the numbers in each sum to r. Then, consider the strategy ϕ which masks the TCs in $\{f_i | s_i \in S_1\}$ and f_{n+1} with $\tilde{f}_1$, and masks the TCs in $\{f_i | s_i \in S_2\}$ and f_{n+2} with $\tilde{f}_2$. It is easy to check that $\tilde{U}_{\tilde{f}_1}(\phi) = \tilde{U}_{\tilde{f}_2}(\phi) = 0 = \tilde{U}^*(\phi)$, making ϕ an optimal strategy. On the other hand, suppose the defender's optimal ϕ yields $\tilde{U}^*(\phi) = 0$. Since $\tilde{f}_1$ must mask f_{n+1}, and $\tilde{f}_2$ must mask f_{n+2}, neither of the OCs are unused. Since $\tilde{U}^*(\phi) = 0$, w.l.o.g., assume $\tilde{U}_{\tilde{f}_1} = 0$. Hence, the sum of utilities of the TCs masked with $\tilde{f}_1$ must be 0. Therefore, the sum of utilities of TCs masked by $\tilde{f}'$ is also 0. Then, $S_1 = \{s_i | \phi_{f_i,\tilde{f}_1} = 1\}$, and $S_2 = \{s_i | \phi_{f_i,\tilde{f}_2} = 1\}$ form a partition of S, each having sum of the elements r. It follows that, *PART* should output *YES* iff *CDG-Robust* finds an optimal strategy ϕ with $\tilde{U}^*(\phi) = 0$. This reduction, being polynomial-time, proves the claim. □

[2] A detailed proof can be found in the online appendix: https://www.dropbox.com/s/n3wn0glm2clzs7e/Appendix.pdf?dl=0

4.2 The Defender's Optimization Problem

The defender's optimal strategy ϕ can be computed by solving the optimization problem given below.

$$\min_{u,\phi} \quad u \tag{6a}$$

$$\text{s.t.} \quad u \sum_{f \in F} \phi_{f,\tilde{f}} \geq \sum_{f \in F} \phi_{f,\tilde{f}} U_f \qquad \forall \tilde{f} \in \tilde{f} \tag{6b}$$

Constraints (1) ~ (4)

The objective function in Equation (6a) minimizes the utility u the adversary receives for the game. Equation (6b) enforces that the adversary chooses a best response to the defender's strategy ϕ, where the expected utility for attacking a given $\tilde{f}$ is given by (5). Constraints (1)~(4) represent a feasible defender strategy.

This optimization problem is non-convex due to constraint (6b), which can be linearized, to convert the optimization problem to an MILP as follows. First, we devise an alternate representation of defender's strategy ϕ, as a $|K| \times |\tilde{F}|$ (0,1)-matrix σ, where $\sigma_{k,\tilde{f}} = 1$ denotes system k is masked with $\tilde{f}$. Further, we represent the TSN N via a vector $\mathbf{x}$, where $x_k \in F$ represents the TC for system k. Then, for each TC f, we have $N_f = |K_f|$ where, $K_f = \{k \in K \mid x_k = F\}$, and $\phi_{f,\tilde{f}} = \sum_{k \in K_f} \sigma_{k,\tilde{f}} \ \forall f, \forall \tilde{f}$. Hence, the alternate representations are indeed equivalent. Then, constraints equivalent to (1)~(4) can be easily formulated for σ and x with an additional constraint $\sum_{\tilde{f} \in \tilde{F}} \sigma_{k,\tilde{f}} = 1 \ \ \forall k \in K$ to ensure feasibility. More importantly, equation (6b) can be reformulated as

$$u \sum_{k \in K} \sigma_{k,\tilde{f}} \geq \sum_{k \in K} \sigma_{k,\tilde{f}} U_{x_k} \quad \forall \tilde{f} \in \tilde{F} \tag{7}$$

The left hand side of (7) can be seen as the sum of a set of terms $u\sigma_{k,\tilde{f}}$, each of which is the product of binary variable $\sigma_{k,\tilde{f}}$ and the continuous variable u. Such an expression can be linearized by introducing variables $z_{k,\tilde{f}}$ for each $k \in K$ and $\tilde{f} \in \tilde{F}$, and enforcing $z_{k,\tilde{f}} = u\sigma_{k,\tilde{f}}$. Consequently, we can rewrite (7) as:

$$\sum_{k \in K} z_{k,\tilde{f}} \geq \sum_{k \in K} \sigma_{k,\tilde{f}} U_{x_k} \tag{8}$$

To enforce $z_{k,\tilde{f}} = u\sigma_{k,\tilde{f}}$, we consider $u \in [U^{min}, U^{max}]$ where $U^{min} = \min_{f \in F} U_f$ and $U^{max} = \max_{f \in F} U_f$. With these bounds on u, we then include the constraints for each z variable in the optimization problem as follows:

$$U^{min} \sigma_{k,\tilde{f}} \leq z_{k,\tilde{f}} \leq U^{max} \sigma_{k,\tilde{f}} \tag{9}$$

$$u - (1 - \sigma_{k,\tilde{f}}) U^{max} \leq z_{k,\tilde{f}} \leq u - (1 - \sigma_{k,\tilde{f}}) U^{min} \tag{10}$$

After this conversion the optimization problem becomes an MILP. The complete formulation can be found in the online appendix.

4.3 Greedy-Minimax Algorithm

Despite the speedup via cut generation, solving the above MILP can still be computationally expensive for large instances. Hence, we seek heuristic algorithms which may be suboptimal but run fast and perform well on average. In this section, we describe a simple approach to sequentially assign OCs to the systems, by greedily minimizing attacker's maximum expected utility for the partially built strategy at each stage. Algorithm 1 gives the pseudo-code.

Algorithm 1: Greedy-Minimax

```
1  minIndCost[] ← (min_f̃ c(f, f̃))_{f∈F}
2  minTotCost ← Σ_f N_f * minIndCost[f]
3  initialize minu*, σ_best
4  For iter = 1 ... numIter
5      K_list[] ← shuffle(K)
6      initialize remB ← B, reqB ← minTotCost
7      initialize σ[], N̄[], Ū[]
8      For i = 1 ... |K|
9          k ← K_list[i], f ← x[k]
10         σ[k] ← GMMAssign(f, σ[], N̄, Ū[])
11         N̄[σ[k]] ← N̄[σ[k]] + 1
12         update(Ū[σ[k]])
13         remB ← remB − c(f, σ[k])
14         reqB ← reqB − minIndCost[f]
15     compute u* = max_f̃ Ū[f̃])
16     update(minu*, u*, σ_best, σ)
17 return σ_best
18 Procedure GMMAssign(f, σ[], N̄, Ū[])
19     initialize newU*[]
20     For f̃ ∈ F̃_f
21         If (reqB − minIndCost[f] + c(f, f̃) > remB) Then
22             Continue
23         σ[k] ← f̃
24         newU*[f̃] ← U*(σ)
25     F̃_best ← argmin_f̃ newU*[f̃]
26     generate f̃_best ~ uniRand(F̃_best)
27 return f̃_best
```

Greedy-Minimax starts by computing for each $f \in F$, the minimum cost of masking f with any feasible OC, and subsequently, the minimum total cost of masking all the systems (Lines 1-2). Next, σ_{best} and $minu^*$ are initialized, which respectively denote the final output strategy of the algorithm and the corresponding utility (Line 3). Subsequently, the algorithm is conducted in a number of iterations. In each iteration, a random shuffle of the set of systems is obtained, referred to as K_{list} above. Subsequently, the strategy σ which is a candidate solution corresponding to this shuffle, the corresponding observed state of the network $(\bar{N}_{\tilde{f}})_{\tilde{f} \in \tilde{F}}$, and the corresponding utilities $(\bar{U}_{\tilde{f}})_{\tilde{f} \in \tilde{F}}$ are all initialized. These are constantly maintained as the algorithm loops through K_{list}, building the solution by assigning an OC to a system one by one (Lines 8-10). The OC to be assigned for a system is determined via the function $GMMAssign()$ which is the essence of this heuristic algorithm. The input to this function is the TC f of the system in question, and the currently built solution in terms of $\sigma, \bar{N}, \bar{U}$. Given these, the function considers the candidate OCs in $\tilde{F}$ one by one, refutes those which lead to violation of the budget constraint (i.e., make the resultant minimum required budget to exceed the resultant remaining

budget). For every other $\tilde{f}$, it computes resultant $\bar{U}_{\tilde{f}}$ if the system is masked with $\tilde{f}$, and stores it in the array $newU^*$ (Lines 19, 23-24). Finally, based on these, it uniformly randomly chooses an OC from those which minimize the resultant utility $newU^*()$ (Lines 25,26). Once *GMMAssign*() returns an OC $\tilde{f}$, it is assigned to the system in question, $\bar{N}_{\tilde{f}}$, $\bar{U}_{\tilde{f}}$ are updated accordingly, as well as the remaining budget and the minimum required (Lines 11-14). Once the loop through K_{list} is over and the full strategy σ is built, its utility utility $u*$ is computed, and compared with $minu^*$, to update $minu^*$ and σ_{best} appropriately (Lines 15-16).

It is possible to conceive examples where this heuristic approach does not yield a good solution on an arbitrary shuffle, even for problem instances with small parameters. Such an example with 4 systems, 4 TCs and 2 OCs is discussed in the online appendix. Further, we also show an example (in the online appendix) where the solution value is $\Theta(|K|)$ times as bad as the optimal, on exponentially many shuffles. This motivates getting candidate solutions for a high number of shuffles and choosing the best among them as described above. Since the greedy choice does not guarantee optimality, we also propose *Soft-GMM*, a slight modification of GMM which makes assignment probabilistically, and not deterministically. It works exactly as GMM, except Lines 25,26 — it draws f_{best} from a distribution $P(\tilde{F})$ where, $P(\tilde{f}) \propto \exp(-newU^*[\tilde{f}])$.

5 OPTIMAL DEFENDER STRATEGY AGAINST NAIVE ADVERSARY

The robust approach to solving CDGs, i.e., assuming a powerful adversary with knowledge of ϕ, can cause the defender to not fully realize the benefit of her informational advantage when faced with a less powerful attacker. In particular, the adversary may value OCs in a fixed manner that is known to the defender.[3] In this case, the values $\bar{U}_{\tilde{f}}$ are fixed and the defender's strategy does not affect the adversary's expected utility for attacking some $\tilde{f}$. Importantly, if there is no budget constraint we can solve for the defender's optimal strategy ϕ in polynomial time using Algorithm 2. W.l.o.g. we assume the adversary has a strict preference ordering over $\tilde{F}$ as if $\bar{U}_{\tilde{f}}$ is equal for any two OCs, the sets could be merged from the defender's perspective, with feasibility constraint and cost adjusted accordingly.

Algorithm 2 begins by initializing ϕ, Γ^* (which stores the TCs the adversary attacks) and $\tilde{f}^*$ (the OC the adversary attacks given ϕ). In Line 3 we compute the matrix $minUtil[]$ which stores the lowest utility achievable for each TC which is $\min_{\tilde{f} \in \tilde{F}_f} \bar{U}_{\tilde{f}}$. The for loop in Line 4 iterates over all $\tilde{f} \in \tilde{F}$ which is sorted descending by $\bar{U}_{\tilde{f}}$ (Line 2) and determines for each $\tilde{f}$ the best set of TCs to mask if $\tilde{f}$ is attacked by the adversary in Lines 5 through 12. To do this, F is split into 4 separate sets P_1, P_2, P_3 and P_4 and the set of TCs to be masked with $\tilde{f}_i$ is stored in Γ'. P_1 contains all TCs which cannot be masked with an $\tilde{f}$ that has $\bar{U}_{\tilde{f}} < \bar{U}_{\tilde{f}_i}$. Intuitively, if this set is non-empty it means the defender is not able to devise a strategy ϕ such that the adversary prefers to attack $\tilde{f}_i$, and hence,

[3] As an example, the adversary could estimate his utility according to values derived from the NIST National Vulnerability Database [21].

Algorithm 2: Compute defender's optimal ϕ with fixed $\bar{U}_{\tilde{f}}$.

1 **initialize** ϕ, Γ^*, $\tilde{f}^*$
2 sort($\tilde{F}$) //descending by utility $\bar{U}_{\tilde{f}}$
3 minUtil[] := $(\min_{\tilde{f}} \bar{U}_{\tilde{f}})_f$
4 **For** $i = 1, \ldots, |\tilde{F}|$
5 **initialize** Γ'
6 P_1:=$\{f \mid minUtil[f] > \bar{U}_{\tilde{f}_i}\}$
7 **If** $P_1 \neq \emptyset$
8 **break**
9 P_2:=$\{f \mid minUtil[f] = \bar{U}_{\tilde{f}_i}\}$
10 P_3:=$\{f \mid minUtil[f] < \bar{U}_{\tilde{f}_i}$ and $\tilde{f}_i \in \tilde{F}_f\}$
11 P_4:=$\{f \mid minUtil[f] < \bar{U}_{\tilde{f}_i}$ and $\tilde{f}_i \notin \tilde{F}_f\}$
12 $\Gamma' := P_2$
13 update(Γ', P_3)
14 update(Γ^*,Γ',$\tilde{f}^*$,$\tilde{f}_i$)
15 update(ϕ,Γ^*,$\tilde{f}^*$)
16 **return** ϕ

all subsequent $\tilde{f}_i$ will never be preferred by the adversary. P_2 (P_4) contain TCs f which must be masked (cannot be masked) with $\tilde{f}_i$. P_3 then contains all TCs f which can be masked with $\tilde{f}_i$ but may also be masked with another OC $\tilde{f}_j \neq \tilde{f}_i$. The function $update(\Gamma', P_3)$ iterates over the TCs $f \in P_3$ and masks all TC f with $\tilde{f}_i \iff U_f \leq EU(\Gamma')$. In Line 13 $update(\Gamma^*, \Gamma', \tilde{f}^*, \tilde{f}_i)$ sets $\Gamma^* = \Gamma'$ and $\tilde{f}^* = \tilde{f}_i$ if $EU(\Gamma') < EU(\Gamma^*)$. Finally, the function $update(\phi, \Gamma^*, \tilde{f}^*)$ in Line 14 determines the OCs $\tilde{f}'$ for all $f \notin \Gamma^*$ given $\bar{U}_{\tilde{f}'} < \bar{U}_{\tilde{f}^*}$ and the strategy ϕ is returned.

Proposition 5.1. *Given fixed utilities $\bar{U}_{\tilde{f}}$ and no budget constraint, Algorithm 2 computes the optimal strategy ϕ in $O(|F||\tilde{F}|)$.* [4]

It is possible to efficiently compute the defender's optimal strategy when there is no budget constraint. When the defender has a budget constraint, however, the question arises if her optimal strategy can be found efficiently as well. We call this problem *CDG-Fixed* and show it to be NP-Hard.

Proposition 5.2. *CDG-Fixed is NP-hard.*

Proof. We prove the proposition via a reduction from the 0-1 Knapsack problem (0-1 KP), which is a classical NP-hard problem. Given a budget B and a set of m items each with a weight w_i and value v_i, 0-1 KP is the optimization problem of finding the subset of items Y which maximizes $\sum_{i \in Y} v_i$ subject to the budget constraint $\sum_{i \in Y} w_i \leq B$. We now show that 0-1 KP can be reduced to *CDG-Fixed*. For convenience, we use $[m]$ to denote the set $\{1, 2..., m\}$ and $S = \sum_{i \in [m]} w_i$ denote the sum of all weights.

Given a 0-1 KP instance as described above, we construct a CDG instance as follows. Let the set of TCs be $F = \{f_1, \ldots, f_m\} \cup \{f_{m+1}\}$, with utilities $U_{f_i} = v_i, \forall i \in [m]$ and $U_{f_{m+1}} = -W$ for some fixed constant W. Let the set of OCs be $\tilde{F} = \{\tilde{f}_1, \tilde{f}_2\}$, with $\tilde{F}_{f_i} = \tilde{F} \, \forall i \in [m]$

[4] The proof can be found in the online appendix.

and $\tilde{F}_{f_{m+1}} = \{\tilde{f}_1\}$. Set the costs as $c(f_i, \tilde{f}_1) = 0$, $c(f_i, \tilde{f}_2) = w_i$ for all $i \in [m]$ and $c(f_{m+1}, \tilde{f}_1) = 0$. Set $\bar{U}_{\tilde{f}_1} > \bar{U}_{\tilde{f}_2}$. Assuming a naive adversary, these components completely define a *CDG-Fixed* problem. Since f_{m+1} is bound to be masked by $\tilde{f}_1$, and $\bar{U}_{\tilde{f}_1} > \bar{U}_{\tilde{f}_2}$, attacking $\tilde{f}_1$ is a dominant strategy for the adversary.

Observe that $\sum_{f \in F} U_f$ is $\sum_{i \in [m]} w_i - W = S - W$. We claim that the optimal objective of the 0-1 KP instance is greater than $S - W$ if and only if the optimal defender utility in the constructed *CDG-Fixed* problem, i.e., $U^*(\phi)$, is negative. We first prove the $\Leftarrow$ direction. Let ϕ^* be the optimal solution to the *CDG-Fixed* problem. By definition, the set $Y = \{i : \phi^*_{f_i, \tilde{f}_2} = 1\}$ is a feasible solution to the 0-1 KP since the cost of mapping f_i to $\tilde{f}_2$ is w_i. The sum of total weights is $S - W$ whereas $U^*(\phi^*) < 0$ meaning the total weights of configurations mapped to $\tilde{f}_1$ is less than 0, this implies that the total weights of configurations mapped to $\tilde{f}_2$ is at least $S - W$. So the optimal objective of the 0-1 KP is also at least $S - W$. The $\Rightarrow$ direction follows a similar argument.

The above claim shows that for any constant W, we can check whether the optimal objective of the 0-1 KP is greater than $S - W$ by solving an *CDG-Fixed* instance. Using this procedure as a black-box, we can perform a binary search to find the exact optimal objective of the 0-1 KP with integer values within $O(poly(\log(S)))$ steps (both S and weights are machine numbers with input size $O(\log(S))$). As a result, we have constructed a polynomial time reduction from computing the optimal objective of any given 0-1 KP to solving the *CDG-Fixed* problem. This implies the NP-hardness of the *CDG-Fixed* problem. □

CDG-Fixed can be solved with Algorithm 2 via a modification to the function $update(\Gamma', P_3)$ in Line 13. Given Γ', we compute the minimum budget B' required to mask all TCs $f \in \Gamma'$ with $\tilde{f}_i$ and mask all TCs $f \in P_3$ and $f \in P_4$ with $\tilde{f}_j$ such that $\bar{U}_{\tilde{f}_j} < \bar{U}_{\tilde{f}_i}$. If $\Gamma' = \emptyset$, then for $f \in P_3$ we mask f with $\tilde{f}_i$ if $c(f, \tilde{f}_i) < B'$. Assuming P_3 is sorted ascending, once the defender assigns $\tilde{f}_i$ to a TC f she is done. If $\Gamma' \neq \emptyset$, the defender must solve at $n = n_{\Gamma'}, \ldots, |K|$ (where $n_{\Gamma'} = |\Gamma'|$) MILPs, given in Problem (11a), to find the best Γ'. Denote $u_{\Gamma'} = EU(\Gamma')$.

$$\min_{\phi} \quad n_{\Gamma'} u_{\Gamma'} + \sum_f \phi_{f,\tilde{f}} U_f \tag{11a}$$

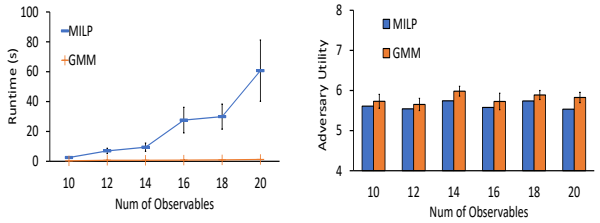
$$\text{s.t.} \quad \sum_f \phi_{f,\tilde{f}_i} \leq n - n_{\Gamma'} \tag{11b}$$

Constraints (1) ~ (4)

6 EXPERIMENTS

We evaluate the CDG model and solution techniques using synthetically generated game instances. The game payoffs are set to be zero-sum, and for each TC, the payoffs U_f are uniformly distributed in $[1, 10]$. Each OC $\tilde{f}$ is randomly assigned a set of TCs it can mask, while ensuring each TC can be masked with at least one OC. To generate a network state $\mathbf{x}$, each system is randomly assigned a TC uniformly at random. The costs $c(f, \tilde{f})$ are uniformly distributed in $[1, 100]$ with the budget B uniformly distributed in-between the minimum cost assignment and maximum cost assignment. All experiments are averaged over 30 randomly generated game instances and have 50 TCs.

6.1 Powerful Adversary - Scalability and Solution Quality Loss



(a) All-powerful Adversary (b) Fixed Utilities

Figure 2: Runtime Comparison and Solution Quality Comparison (15 Systems) - MILP and Greedy MaxiMin (GMM) with 1000 random shuffles.

When solving for the defender's optimal strategy ϕ strategy for enterprise networks, it is important to have solution techniques which can scale to large instances of CDGs. Our first experiment compares the scalability of the MILP to the Hard Greedy Minimax (GMM) algorithm with 1000 random shuffles and the solution quality of the two approaches. In Figure 2(a) we show the runtime results with the runtime in seconds on the y-axis and the number of OCs varied on the x-axis. The runtime for solving the MILP increases exponentially as the number of OCs increases while GMM finishes in under 1 seconds in all cases.

While GMM is much faster than the MILP, it is not guaranteed to provide the optimal solution. However, our experimental results show that empirically the solution quality loss is very small. In Figure 2(b) we compare the solution quality of the MILP to GMM, where the attacker's utility is given on the y-axis and the number of OCs are varied on the x-axis. Importantly, GMM shows a low solution quality loss for the defender compared to the MILP with a minimum loss of 1.76% for 12 OCs and a maximum loss of 3.40% for 16 OCs. This experiment highlights the scalability of GMM and show the loss in solution quality from GMM gives a reasonable trade-off between computational efficiency and solution quality.

An interesting feature of GMM is how often it returns the optimal solution for the defender as the CDG game size changes. Table 1 compares the solution quality of GMM (with 1000 random shuffles) versus the MILP for several game sizes with 10 and 20 systems where the number of OCs are varied from 2 to 10. Interestingly, for CDGs with 10 systems, Hard-GMM is able to find the optimal solution in a vast majority of instances (142 out of 150 instances). However, for CDGs with 20 systems, GMM fails to recover the optimal solution in about a third of the instances (96 out of 150). Nevertheless, the loss of solution quality still remains low (3.18%) even when GMM returns the optimal solution a third of the time.

We also tested the solution quality of a variation of GMM, called GMM−λ. Instead of greedily choosing the OC with minimax expected utility at the stage, we apply a soft-min function [9] with

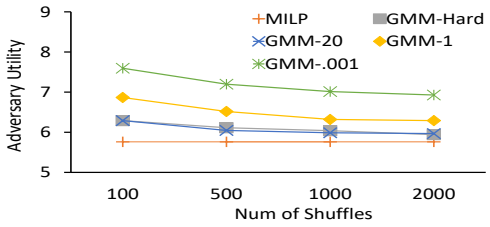


Figure 3: Solution Quality Comparison (20 systems and 15 OCs) - MILP and GMM varying the number of shuffles used for the GMM.

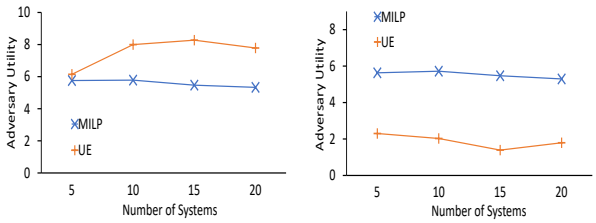
# OCs	2	4	6	8	10
10 systems	0	0.092%	0.015%	0.028%	0.512%
Optimal Instances	30	29	29	29	25
20 systems	0	0.028%	0.615%	1.91%	3.18%
Optimal Instances	30	28	17	12	9

Table 1: Solution Quality % loss and number of optimal instances for GMM versus MILP.

parameter λ controlling the greediness of choice. Figure 3 shows the solution quality of the MILP versus GMM (denoted as GMM-Hard) and GMM–λ with varying λ value. GMM–.001 is very close to randomly choosing OCs for the systems and it performs poorly with the limited number of shuffles, indicating that the GMM is an effective heuristic. While not clearly seen in this set of experiments, the randomness in GMM–λ leads to a potential of finding better strategies than GMM since GMM-Hard is restricted to a limited strategy space and GMM–λ is not. We defer further investigation to future work.

6.2 Comparing Solution for Different Types of Adversaries

Our last experiment compares how the optimal strategies for the two adversary models (powerful versus naive) perform in the opposite case. Figure 4(a) compares the solution quality of the MILP in Section 4 to Algorithm 2 when the adversary is assumed to know ϕ with the attacker's utility on the y-axis and the number of systems varied on the x-axis. This figure highlights that for the powerful adversary the MILP performs significantly better than Algorithm 2 (except for 5 systems) and shows the importance of considering the adversary's information when devising the defender's strategy ϕ. In Figure 4(b) we compare the solution quality of Algorithm 2 to the MILP when the adversary is assumed have fixed utilities. As the figure shows, the improvement in utility is dramatically higher for Algorithm 2 compared to the MILP. The reason for this difference lies in Algorithm 2 leveraging the fact the adversary has a fixed preference over OCs and minimizes the value of systems masked with the OC the adversary will attack. The MILP, however, minimizes the worst case utility given the adversary may attack any OC and hence, fails to leverage the defender's advantage to a high benefit.



(a) Powerful Adversary (b) Naive Adversary

Figure 4: Solution Quality Comparison (10 OCs) - In (a) we show the solution quality of two strategies (one computed by MILP in Section 4, one computed by Algorithm 2) against a powerful adversary. In (b) we show the solution quality of the strategies against a naive adversary.

7 CONCLUSION

In this paper, we study the problem of a network administrator should respond to scan requests from an adversary attempting to infiltrate her network. We show that computing the optimal defender strategy against a powerful adversary is NP-hard and provide an MILP to solve for their optimal strategy. Additionally, we provide a greedy algorithm which quickly finds good defender strategies and performs well empirically. We then show that computing the optimal strategy against a naive attacker is still NP-hard given a budget constraint. Finally, we give extensive experimental analysis demonstrating the effectiveness of our approaches.

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8 APPENDIX

8.1 Missing Proofs

Proof of Proposition 4.1.

Proof. Equivalently, we show that, $\tilde{U}^*(\phi) \geq \frac{\sum_{f\in F} N_f U_f}{|K|}$ for all ϕ. Fix any $\phi \in \Phi$. We have,

$$\tilde{U}^*(\phi) \geq \tilde{U}_{\tilde{f}}(\phi) \quad \forall \tilde{f} \qquad \text{(by definition of } \tilde{U}^*(\phi))$$

$$\therefore \quad \sum_{\tilde{f}\in\tilde{F}} N_{\tilde{f}} \tilde{U}^*(\phi) \geq \sum_{\tilde{f}\in\tilde{F}} N_{\tilde{f}} \tilde{U}_{\tilde{f}}$$

$$\therefore \quad |K| \cdot \tilde{U}^*(\phi) \geq \sum_{\tilde{f}\in\tilde{F}} \sum_{f\in F} \phi_{f,\tilde{f}} U_f \qquad \text{(using (5))}$$

$$= \sum_{f\in F} \left(U_f \sum_{\tilde{f}\in\tilde{F}} \phi_{f,\tilde{f}} \right) \qquad \text{(re-ordering terms)}$$

$$= \sum_{f\in F} U_f N_f \qquad \text{(by definition of } \phi, N_f)$$

$$\therefore \quad \tilde{U}^*(\phi) \geq \frac{\sum_{f\in F} N_f U_f}{|K|}$$

Since the choice of ϕ was arbitrary, the claim follows. □

Proof of Proposition 6.1.

Proof. We first show that for each $\tilde{f} \in \tilde{F}$, Lines 5 through 13 in Algorithm 2 computes the set Γ' with the minimum average value. To see this, note that all TCs $f \in P_2$ must be in Γ' while all TCs $f \in P_4$ cannot be included. In $update(\Gamma', P_3)$ (note P_3 is given in sorted order) the defender decides for each $f \in P_3$ to include the N_f TCs in $\Gamma' \iff U_f \leq EU(\Gamma')$. At the end of this update, tt follows that Γ' must be the minimum average set for $\tilde{f}_i$. Given that the for loop in Line 4 iterates through all $\tilde{f} \in \tilde{F}$, it must be the case that the optimal Γ^* is returned for some $\tilde{f}$.

In Line 2, sorting $\tilde{F}$ takes $O(|\tilde{F}|\log|\tilde{F}|)$ time and calculating $minUtil[]$ takes $O(|F||\tilde{F}|)$ time. For each iteration of the for loop in Line 4, it takes $O(|F|)$ time to split F into the sets the four sets P_1, P_2, P_3 and P_4. It takes the function $update(\Gamma', P_3)$ at most $|F|$ operations to update Γ' while $update(\Gamma^*, \Gamma', \tilde{f}^*, \tilde{f}_i)$ takes $O(1)$ time. Hence, each iteration it takes $O(|F|)$ time and hence, $O(|F||\tilde{F}|)$ time for the for loop. Lastly, $update(\phi, \Gamma^*, \tilde{f}^*)$ takes at $O(|F||\tilde{F}|)$ time to return the defender's strategy ϕ as it must find an OC $\tilde{f}_j$ for each $f \notin \Gamma^*$ with $\bar{U}_{\tilde{f}_j} < \bar{U}_{\tilde{f}_i}$. □

8.2 Full Formulation of MILP for 6a

$$\min_{u,\sigma,z} \quad u \tag{12a}$$

$$\text{s.t.} \quad \sum_{k\in K} z_{k,\tilde{f}} \geq \sum_{k\in K} \sigma_{k,\tilde{f}} U_{x_k} \qquad \forall \tilde{f} \in \tilde{f} \tag{12b}$$

$$\sum_{\tilde{f}\in\tilde{F}} \sigma_{k,\tilde{f}} = 1 \qquad \forall k \in K \tag{12c}$$

$$\sigma_{k,\tilde{f}} \leq \pi_{x_k,\tilde{f}} \qquad \forall k \in F, \forall \tilde{f} \in \tilde{F} \tag{12d}$$

$$\sum_{\tilde{f}\in\tilde{F}} \sum_{k\in K} \sigma_{k,\tilde{f}} c(x_k, \tilde{f}) \leq B \tag{12e}$$

$$U^{min}\sigma_{k,\tilde{f}} \leq z_{k,\tilde{f}} \leq U^{max}\sigma_{k,\tilde{f}} \qquad \forall k \in F, \forall \tilde{f} \in \tilde{F} \tag{12f}$$

$$u - (1-\sigma_{k,\tilde{f}})U^{max} \leq z_{k,\tilde{f}} \qquad \forall k \in F, \forall \tilde{f} \in \tilde{F} \tag{12g}$$

$$z_{k,\tilde{f}} \leq u - (1-\sigma_{k,\tilde{f}})U^{min} \qquad \forall k \in F, \forall \tilde{f} \in \tilde{F} \tag{12h}$$

$$\sigma_{k,\tilde{f}} \in \{0,1\} \qquad \forall k \in F, \forall \tilde{f} \in \tilde{F} \tag{12i}$$

8.3 GMM Examples

Note that the adversary's utility $U^*(\phi)$ for any strategy ϕ can be at most $|K|$ times the optimal value $min_\phi U^*(\phi)$. This follows from observing that for any strategy ϕ, we have $\tilde{U}_{\tilde{f}} \leq \max_{f|N_f>0} U_f \forall \tilde{f}$ by definition, and thus, $U^*(\phi) \leq \max_{f|N_f>0} U_f$, whereas $min_\phi U^*(\phi)$ is at least the average of all the system utilities by Proposition 4.1. Since any choice a greedy heuristic makes can be potentially suboptimal, one may intuitively expect its performance to be worse for a higher number of choices to be made, that is, for larger sized inputs, and relatively better for smaller inputs. However, we show an example instance of a CDG where despite the input size ($|F|, |K|, |\tilde{F}|$) being very small, the (hard-)GMM algorithm in a particular iteration (i.e., when conducted on a particular shuffle of the systems) gives a highly suboptimal solution.

Consider the set of systems $K = \{k_1, k_2, k_3, k_4\}$, the set of TCs $F = \{f_1, f_2, f_3, f_4\}$ and the set of OCs $\tilde{F} = \{\tilde{f}_1, \tilde{f}_2\}$. Let the feasibility constraints be given via the sets $F_{\tilde{f}_1} = \{f_1, f_2, f_3\}$ and $F_{\tilde{f}_2} = \{f_2, f_3, f_4\}$. Let each system k_i have the TC f_i, so that the TSN $(N_f)_{f\in F}$ is $(1, 1, 1, 1)$. For the TCs, let the utilities be $U_{f_1} = 1$, $U_{f_2} = 2$, $U_{f_3} = 30$, and $U_{f_4} = 40$. For simplicity, let all the costs $c(f, \tilde{f})$ to be 0, so that there is essentially no budget constraint.

Consider the ordering of the systems on which GMM is performed to be: $\{k_1, k_2, k_3, k_4\}$. Then, the strategy σ computed by the GMM on this ordering is as follows:

$$\begin{array}{c} \\ k_1 \\ k_2 \\ k_3 \\ k_4 \end{array} \begin{array}{c} \begin{array}{cc} \tilde{f}_1 & \tilde{f}_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

Accordingly, we have the expected utilities of OCs $\tilde{U}_{\tilde{f}_1} = (1 + 2 + 30)/3 = 11$ and $\tilde{U}_{\tilde{f}_2} = 40/1 = 40$, and thus, adversary's utility is 40 for this strategy. The optimal solution, however, masks k_1, k_3 with $\tilde{f}_1$ and k_2, k_4 with $\tilde{f}_2$ giving the expected utilities of the OCs: $\tilde{U}_{\tilde{f}_1} = (40 + 2)/2 = 21$ and $\tilde{U}_{\tilde{f}_2} = (30 + 1)/2 = 15.5$, thus, the optimal being just 21.

Further, the following is an example of a CDG which shows the GMM algorithm can perform $\Theta(|K|)$ as bad as the optimal solution on exponentially many shuffles.

Consider the CDG instance with the set of systems $K = \{k_1, \ldots, k_m\}$, so that $|K| = m$. Let the set of TCs $F = \{f_1, f_2, f_3\}$ and the set of OCs $\tilde{F} = \{\tilde{f}_1, \tilde{f}_2\}$. Let the true state of the network be: $\mathbf{x} = (1, 2, 3, \ldots)$ Let the feasibility constraints be given by the sets $F_{\tilde{f}_1} = \{f_1, f_3\}$ and $F_{\tilde{f}_2} = \{f_2, f_3\}$. For the TCs, the utilities are $U_{f_1} = 1$, $U_{f_2} = 2000$,

and $U_{f_3} = \epsilon$. For simplicity, let all the costs $c(f, \tilde{f})$ to be 0, so that there is essentially no budget constraint.

The optimal solution to this CDG is to assign systems $k_2, \ldots, k_m$ to be masked by $\tilde{f}_2$ with k_1 being masked with $\tilde{f}_1$. This gives the following expected utilities: $\tilde{U}_{\tilde{f}_1} = 1/1 = 1$ and $\tilde{U}_{\tilde{f}_2} = \frac{2000+(m-2)\epsilon}{m-1} = \frac{2000}{m-1} + \frac{(m-2)\epsilon}{m-1}$. Consider any shuffle which orders the systems such that k_1 is first and k_2 is last (of which there are $(m-2)!$). Given any ordering of this type, GMM assigns assigns systems $k_3, \ldots, k_m$ to be masked with $\tilde{f}_1$ and would assign k_2 to be masked with $\tilde{f}_2$. The expected utilities given this assignment is the following: $\tilde{U}_{\tilde{f}_1} = \frac{1+(m-2)\epsilon}{m-1} = \frac{1}{m-1} + \frac{(m-2)\epsilon}{m-1}$ and $\tilde{U}_{\tilde{f}_2} = 2000/1 = 2000$. The loss in this case is $\approx \frac{2000}{\frac{2000}{m-1}} = \frac{1}{m-1}$ which is a $\Theta(|K|)$ loss.