Your friends are better than you: Friendship Paradox and its social consequences

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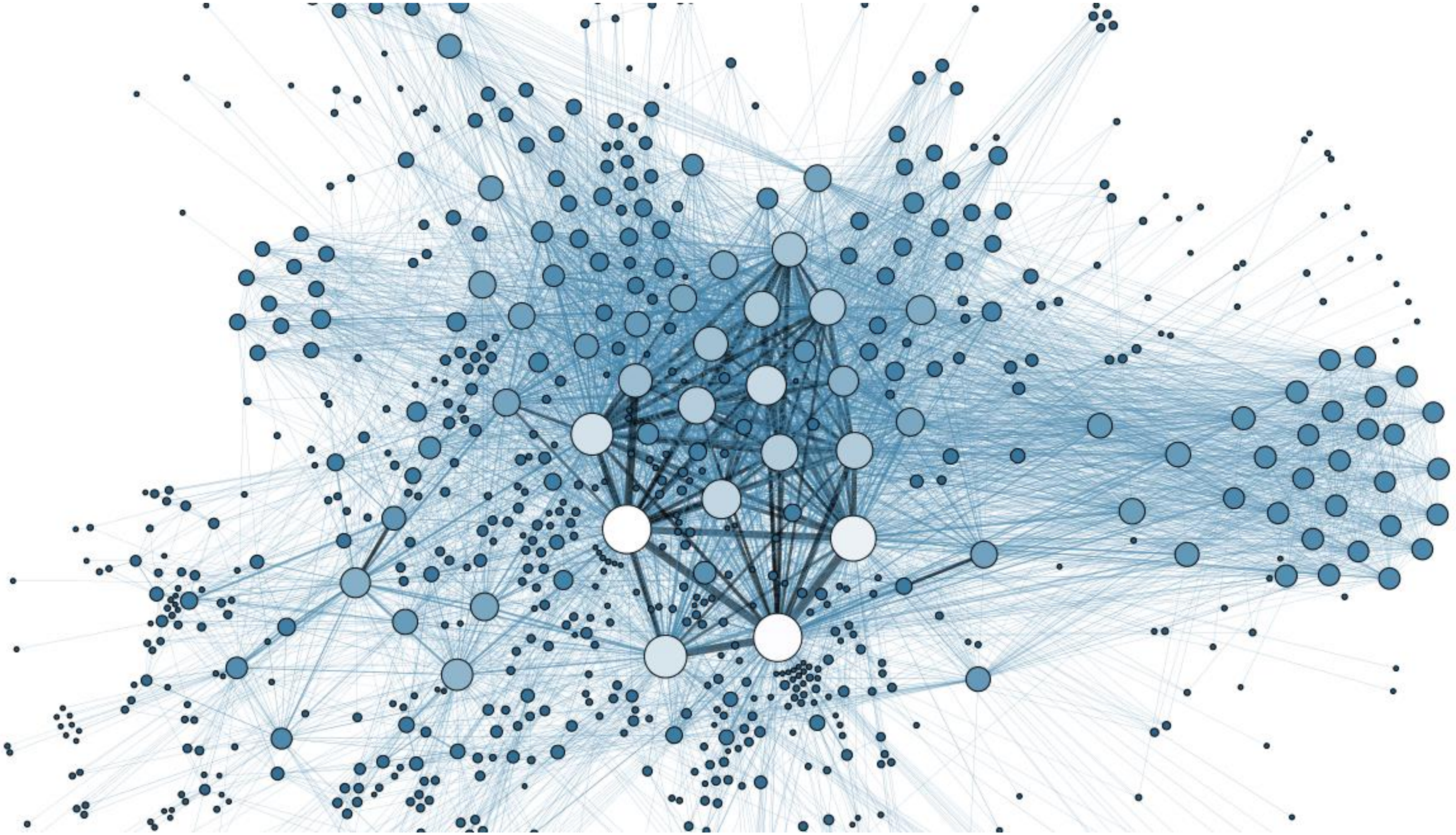
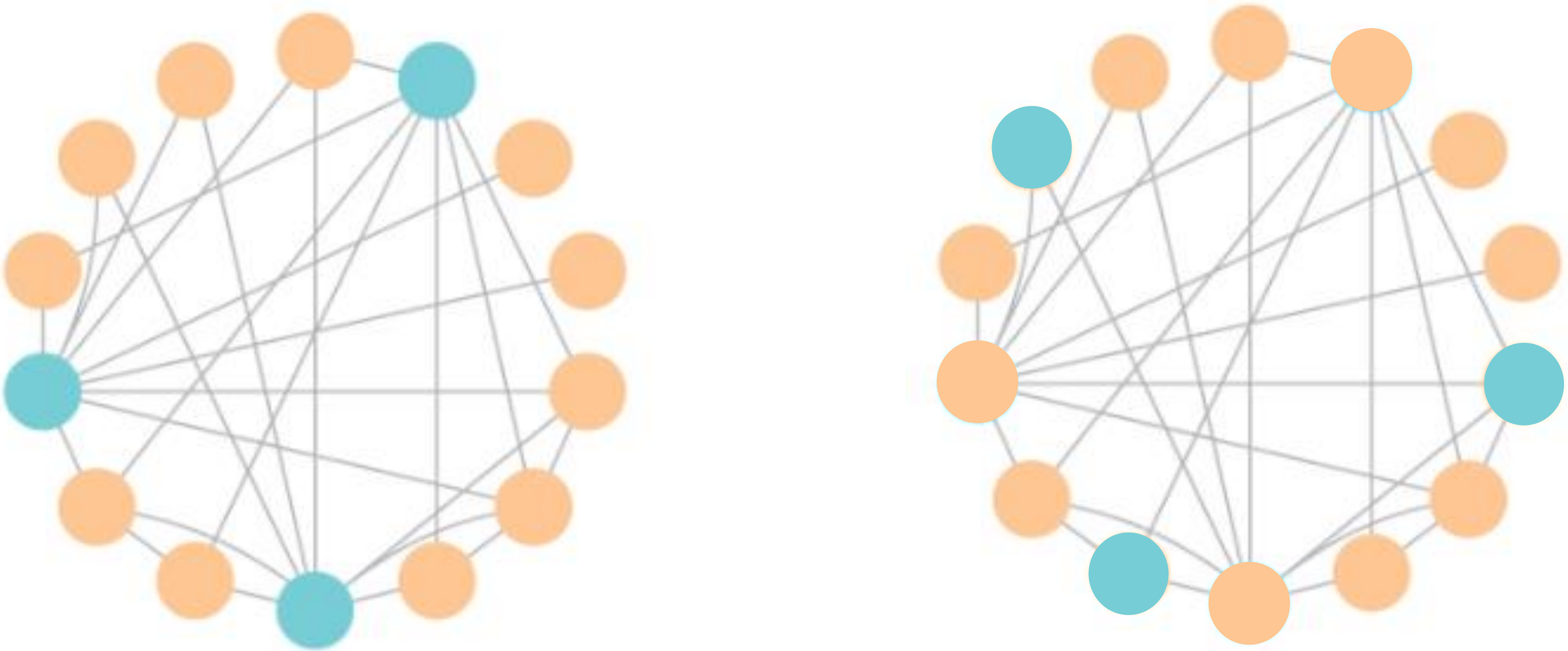
Networks shape behavior and perceptions



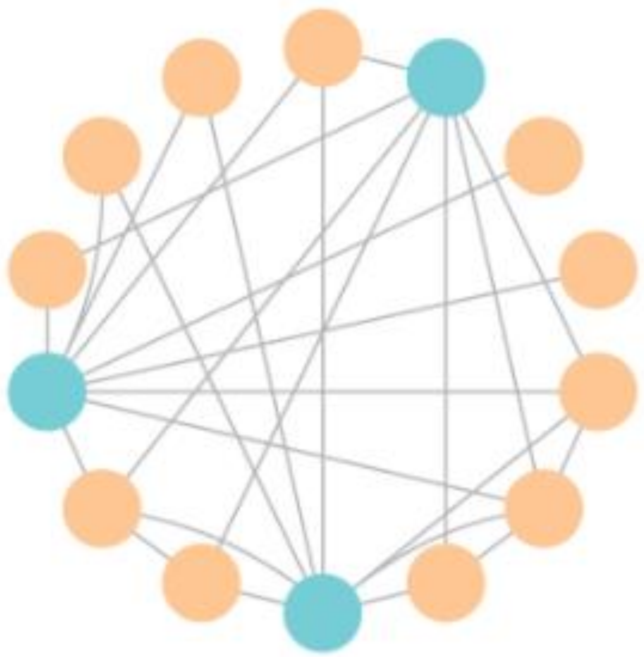
Networks shape behavior and perceptions

1. Networks shape behavior: form the substrate for social interactions and information flow.
2. Your friends are a small subset of the population.
3. Friends are not a random sample of the population.
4. This distorts your perceptions. (A rare trait can appear very popular)

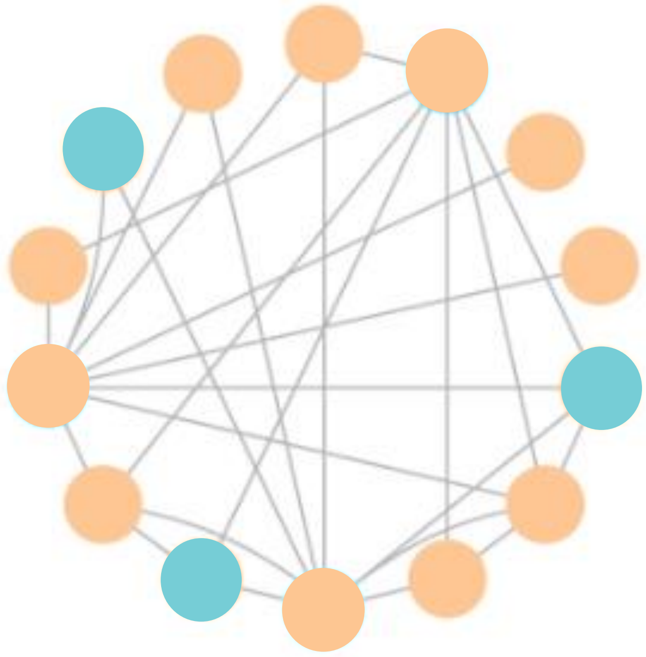
What color is more popular?



What color is more popular?



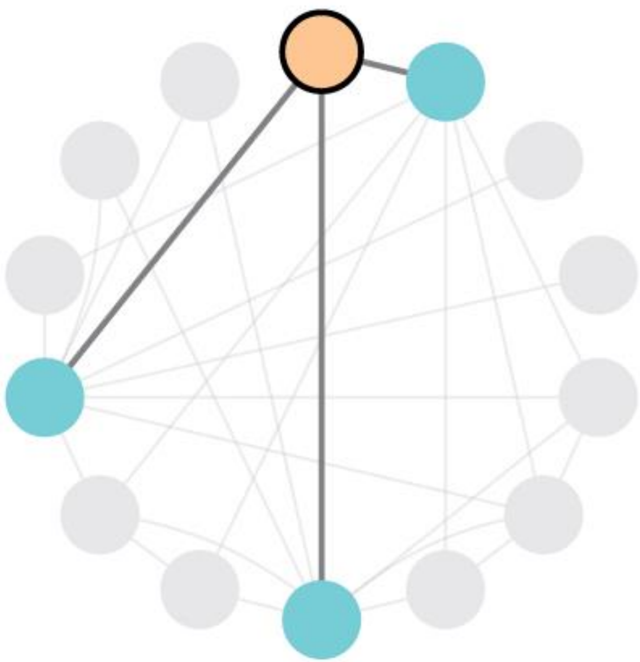
Most nodes think blue is popular

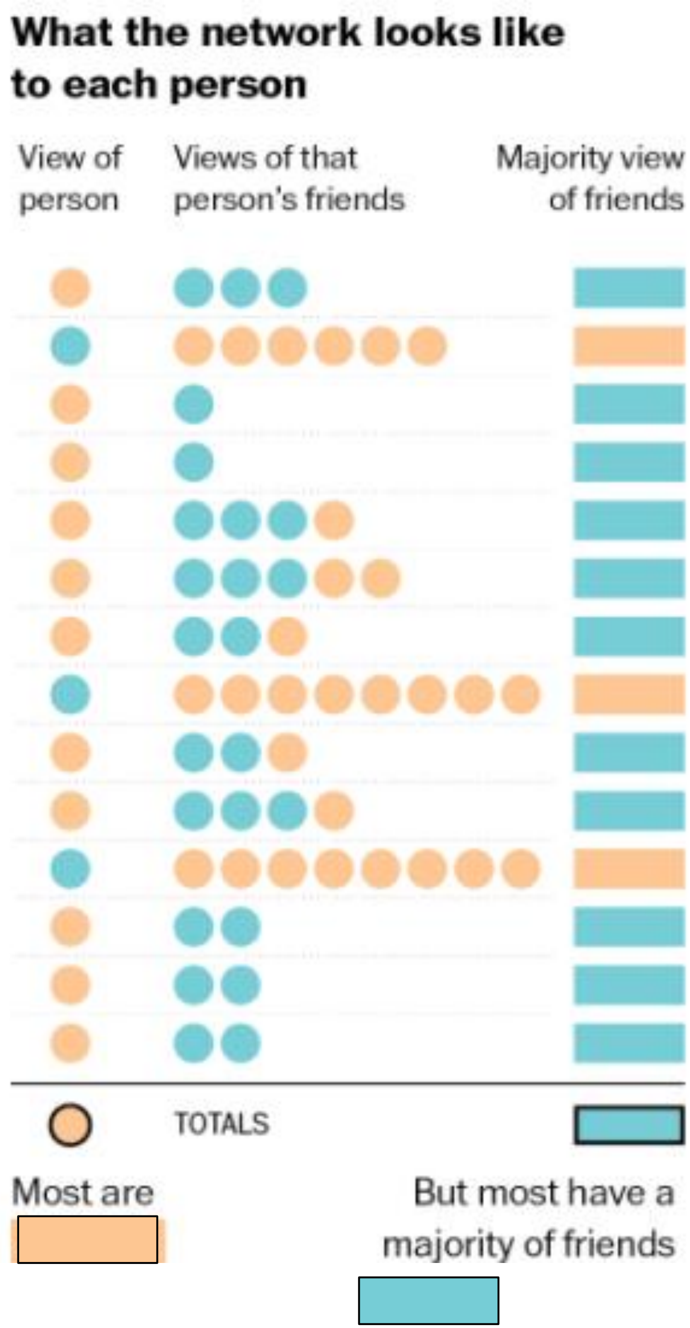


Nodes think yellow is more Blue is not especially popular

Local vs global views

100% of this person's friends think baseball caps are trendy.

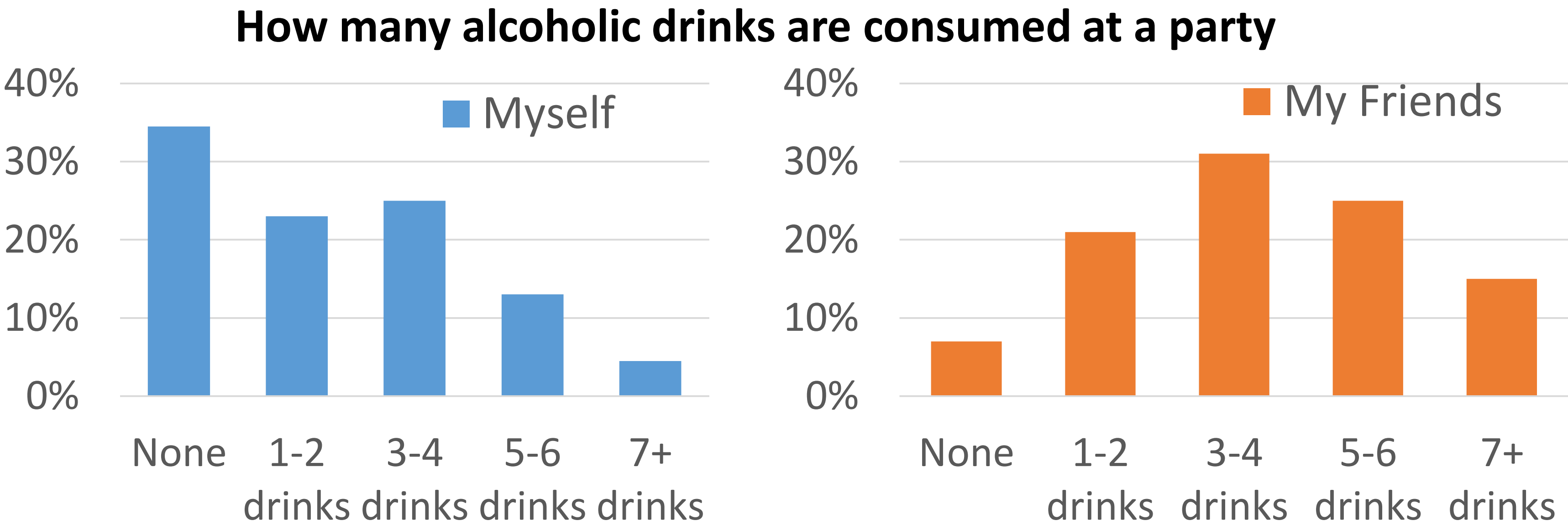




K. Schaul, "A quick puzzle to tell whether you know what people are thinking", *Wonkblog, Washington Post*
https://www.washingtonpost.com/graphics/business/wonkblog/ma

Networks distort local information

- Networks can systematically bias individual perceptions of what is common among peers
  - Example: College students overestimate peers’ alcohol use



Source: Most Students Do PartySafe@Cal

Outline

Friendship paradox

- The many friendship paradoxes in networks
- Origins of paradoxes: a network science view

Perception bias

- Friendship paradox in directed networks
- … biases perceptions of popularity
- Twitter case study: Some hashtags appear more popular than they are

Polling

- Estimate global popularity with perception bias
- … with a limited budget

Lots of paradoxes!

Friendship paradox

You friends have more friends than you do, on average [Feld, 1991]

Lots of paradoxes!

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Generalized friendship paradox

You friends are more *X* than you are, on average [Hodas et al., 2013, Eom & Jo, 2014]

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Strong friendship paradox

Most of your friends have more friends than you do [Kooti et al., 2014]

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Majority illusion

Most of your friends have a trait, even when it is rare. [Lerman et al, 2016]

How common is strong friendship paradox?

Very common… almost everyone observes that most of their friends are more popular

Network	Type	Nodes	Probability of paradox
LiveJournal	Social	3,997,962	**84**
Twitter	Social	780,000	**98**
Skitter	Internet	1,696,415	**89**
Google	Hyperlink	875,713	**77**
ProsperLoan	Social Finance	89,269	**88**
ArXiv	Citation	34,546	**79**
WordNet	Semantic	146,005	**75**

But wait, there is more

Activity

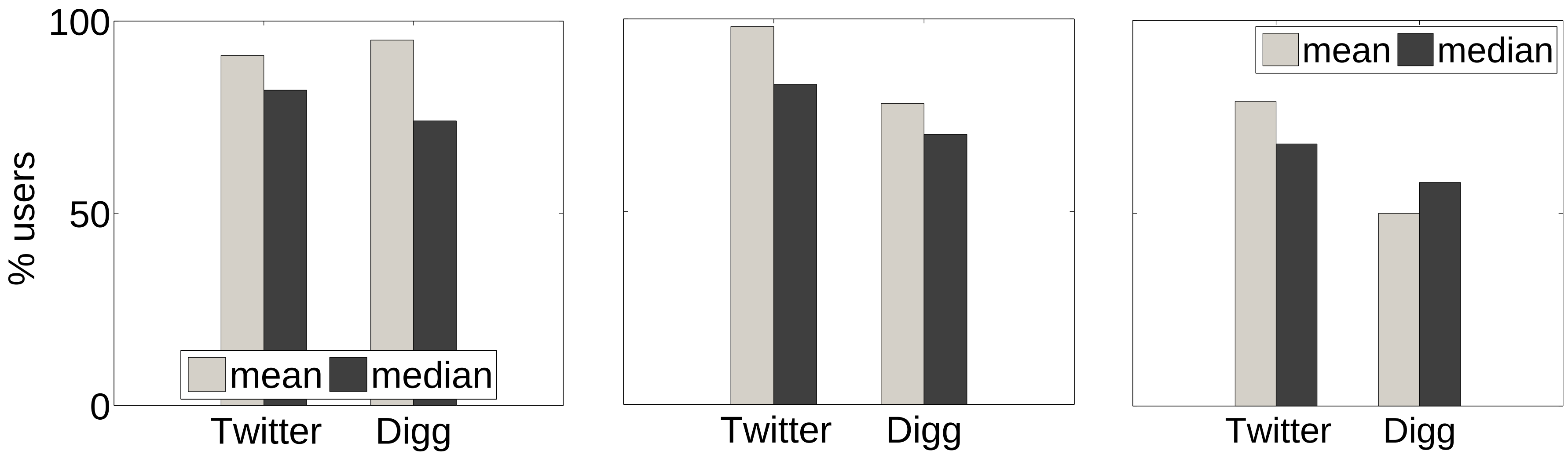
You post less than most of your friends

Diversity

You see less diverse content than most of your friends

Virality

You see less viral content than most of your friends



[Kooti, et al (2014) "Network Weirdness: Exploring the origins of network paradoxes" in *ICWSM*]

Why?

Friendship paradox

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Different explanations

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STATISTICAL SAMPLING EXPLANATION

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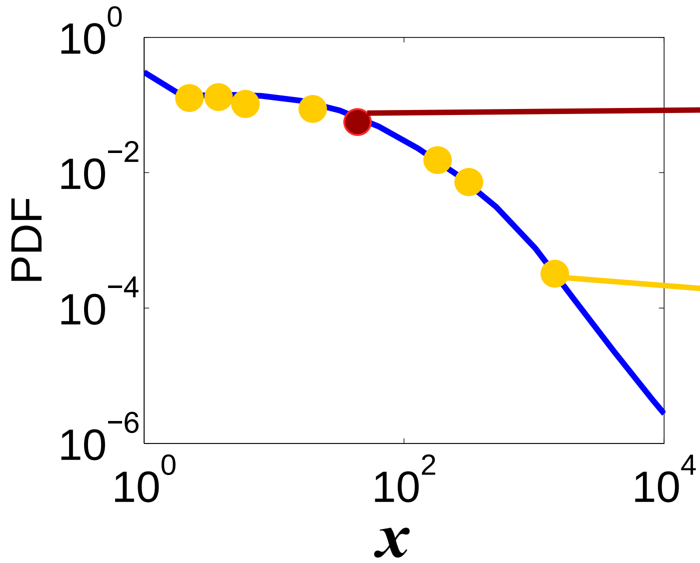
HIGHER ORDER NETWORK STRUCTURE

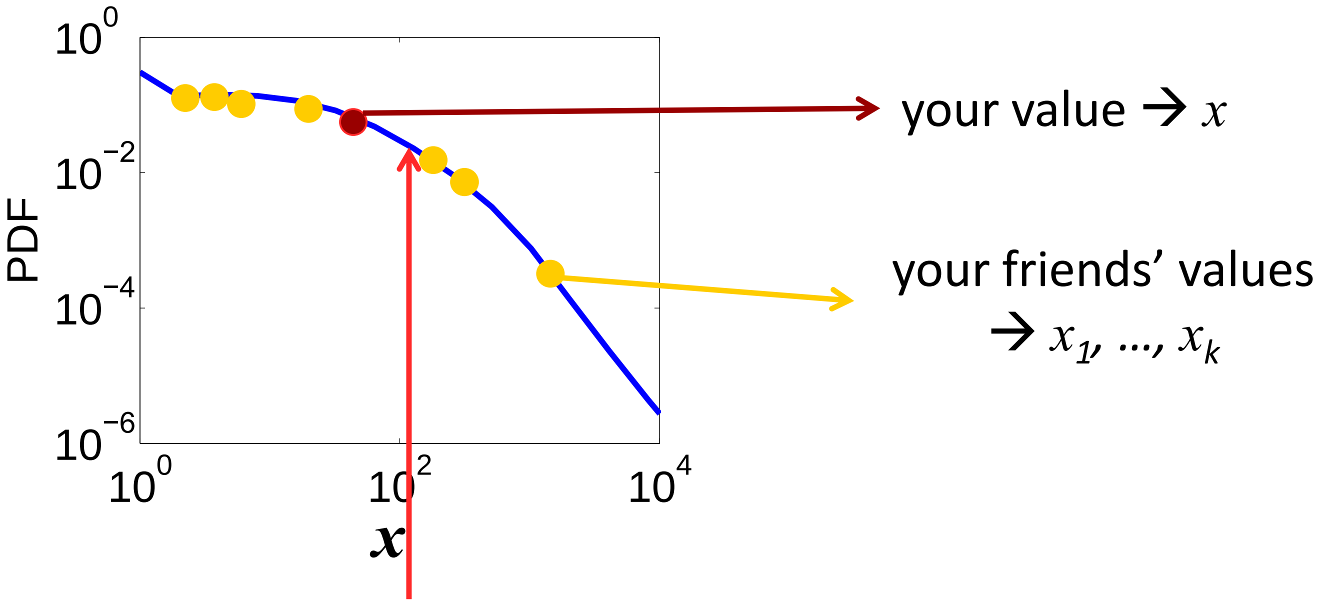
Friendship paradox as a byproduct of sampling from a heterogeneous distribution



your value → x

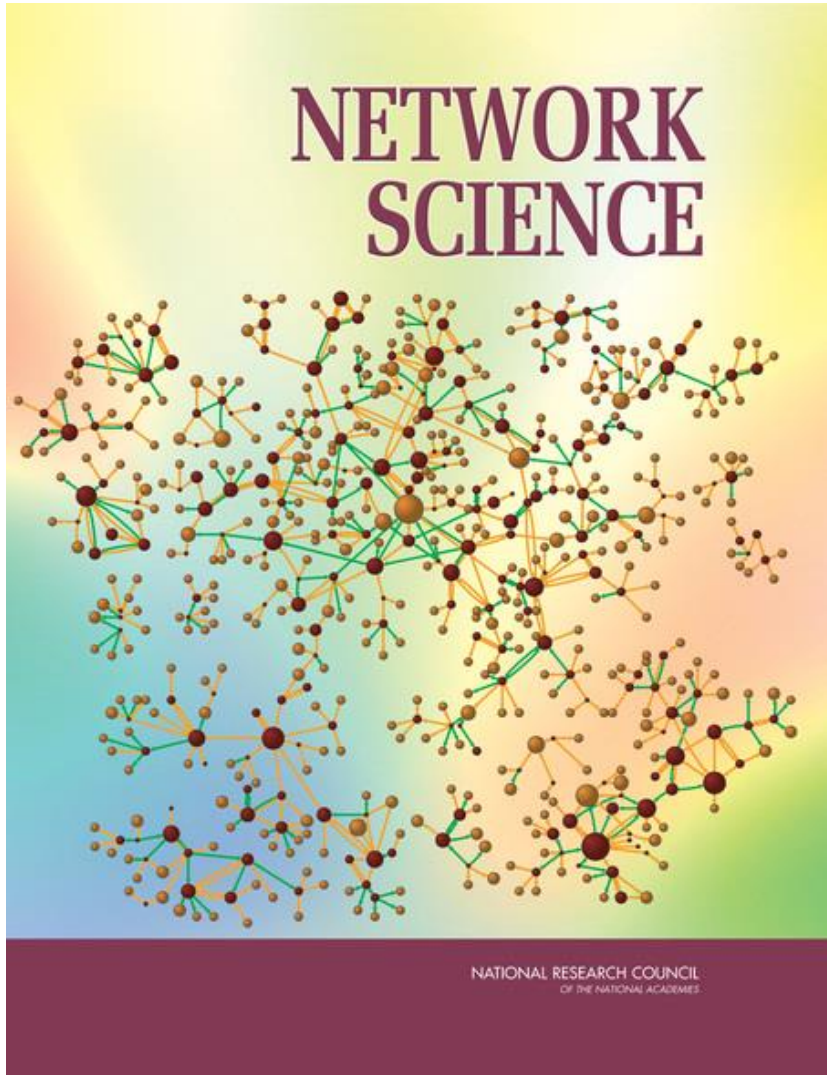
your friends' values → $x_1, \ldots, x_k$

Friendship paradox as a byproduct of sampling from a heterogeneous distribution



$$\textbf{mean} = \frac{1}{k}(x_1 + \cdots + x_k) > x$$

Sample mean grows with *k*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Strong friendship paradox is a network effect

- To explain strong friendship paradox, need to account for network structure
- Building blocks of network structure
- dK series framework represents network structure as the joint degree distribution of subgraphs of up to *d* nodes

P. Mahadevan, D. Krioukov et. al., *ACM SIGCOMM Comp. Comm. Rev*. 36 135–146 (2006)

First-order structure (1K)

- Node degree distribution

Second-order structure (2K)

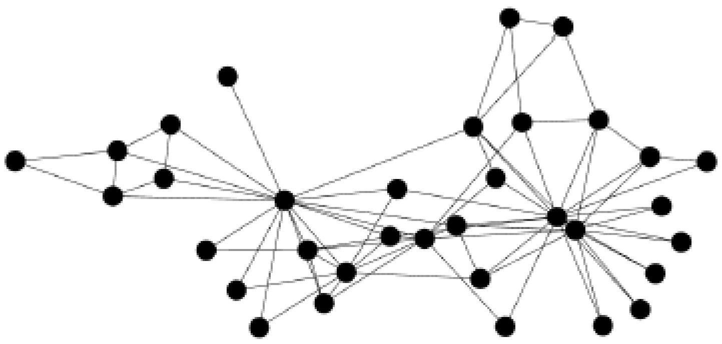
- Pair degree distribution

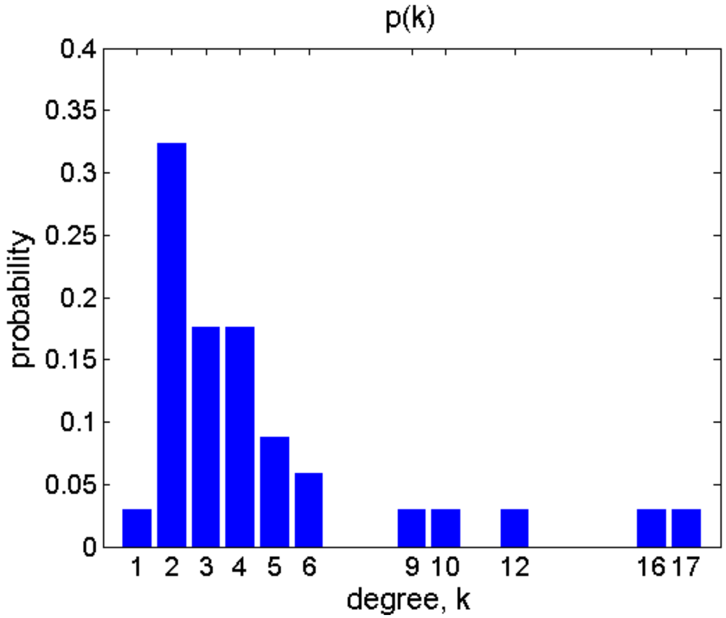
Third-order structure (3K)

- Triplet degree distribution

First-order (1K) structure

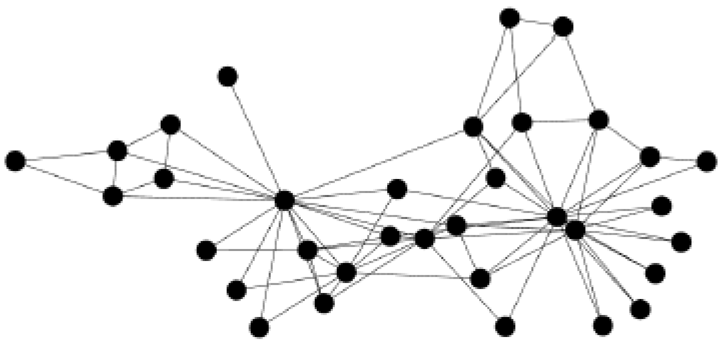
- Node degree distribution $p(k)$
  - Probability that a randomly selected *node* has degree k.
- Any heterogeneous degree distribution (variance > 0) will lead to a (weak) friendship paradox

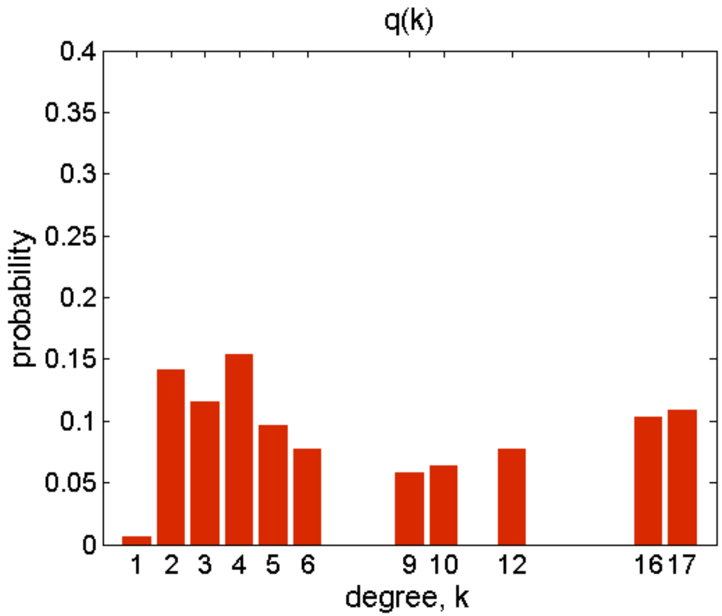




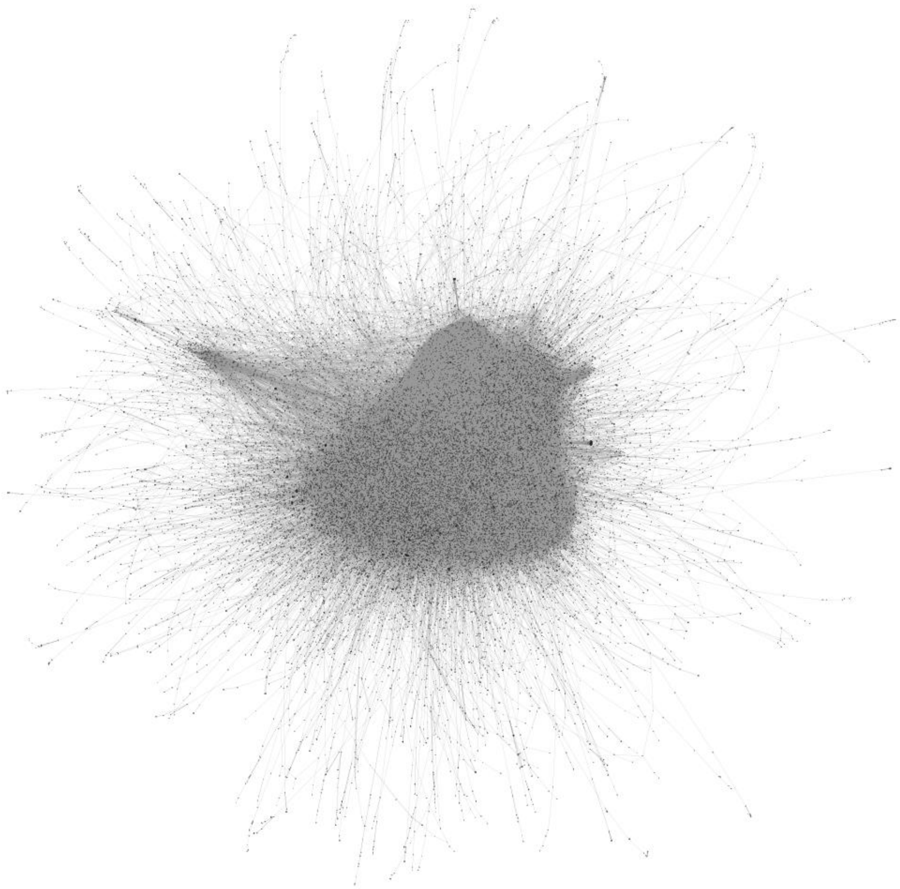
First-order (1K) structure

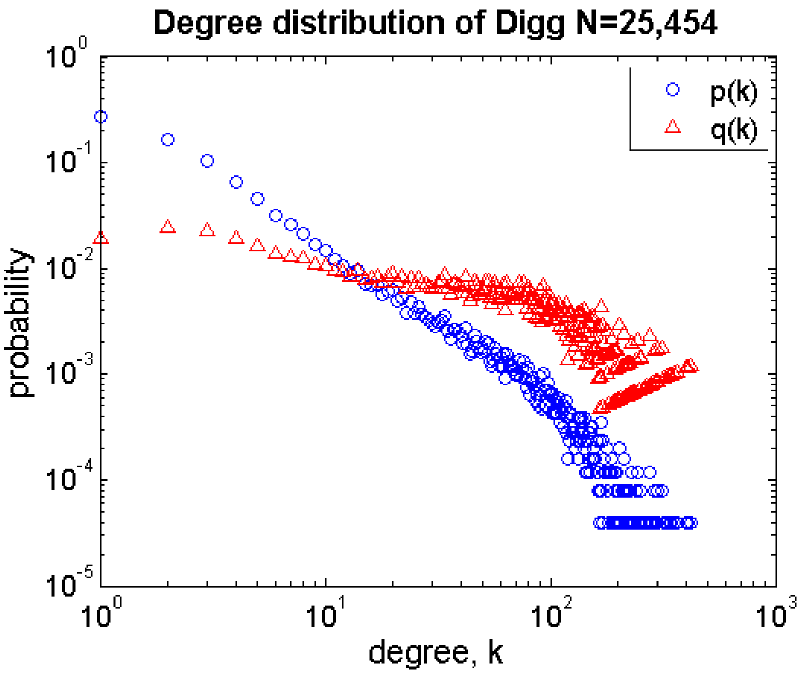
- Neighbor degree distribution $q(k) \sim kp(k)$
  - Probability that a randomly selected *neighbor* has degree k.
- Any heterogeneous degree distribution (variance > 0) will lead to a (weak) friendship paradox





Digg social network

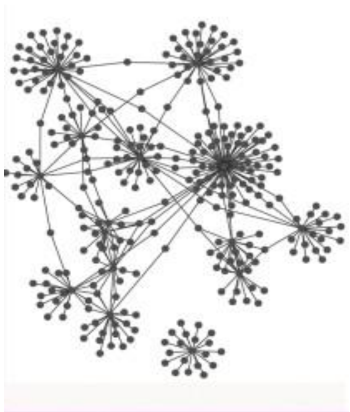




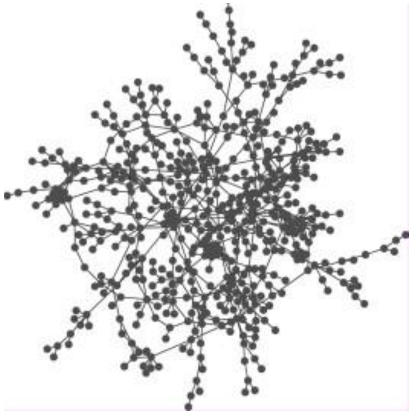
Second-order (2K) structure

- … Nodes do not link at random
- Joint degree distribution of connected pairs of nodes $e(k,k')$
  - Probability that a randomly selected *edge* links nodes with degrees k and k'.
- **Degree assortativity r_{2k}**
- MEJ Newman, Assortative Mixing in Networks, *Phys Rev Lett* 89 208701 (2002)

disassortative



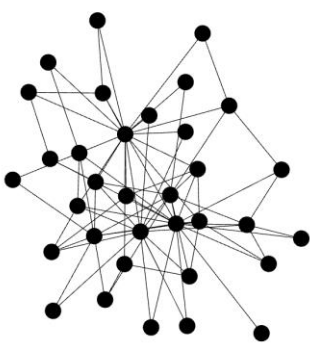
assortative



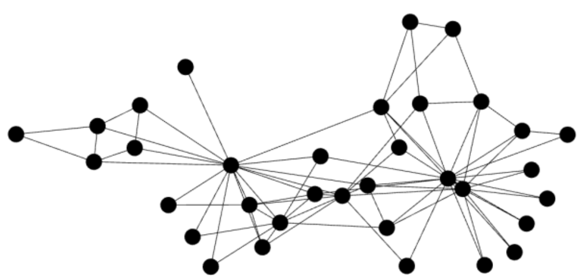
Third-order (3K) structure

- … nodes do not link to random neighbors
- **Neighbor assortativity:** neighbors tend to have similar (or dissimilar) degrees, $\mathbf{r_{3k}}$
- Networks can have the same 1K and 2K structure but different 3K structure
  - Wu, Percus & Lerman, Neighbor Degree Assortativity in Networks, *in preparation*

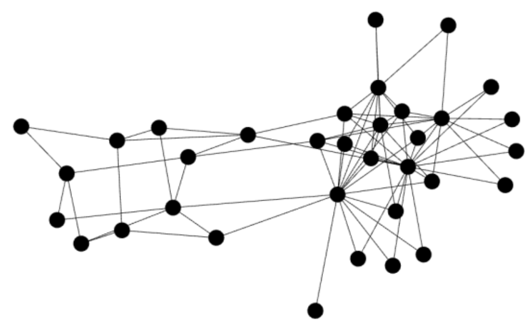
negative neighbor assortativity



original network

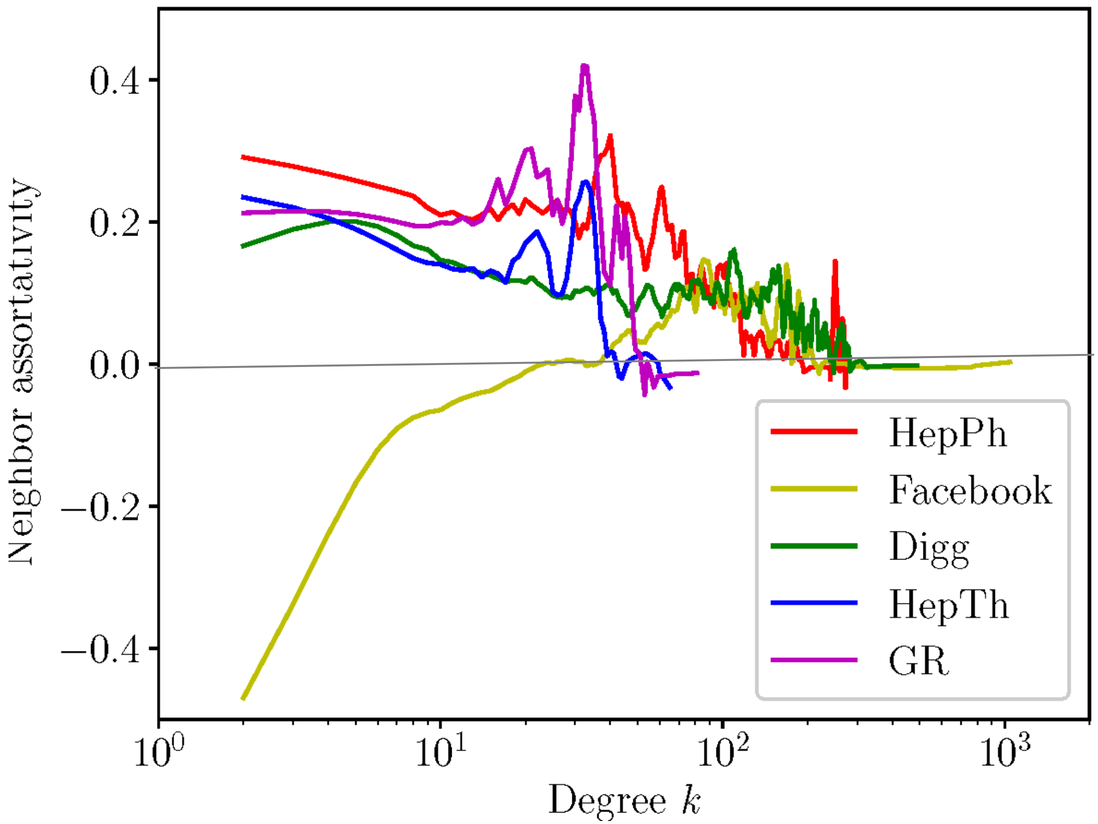


positive neighbor assortativity



Real-world networks have third-order structure

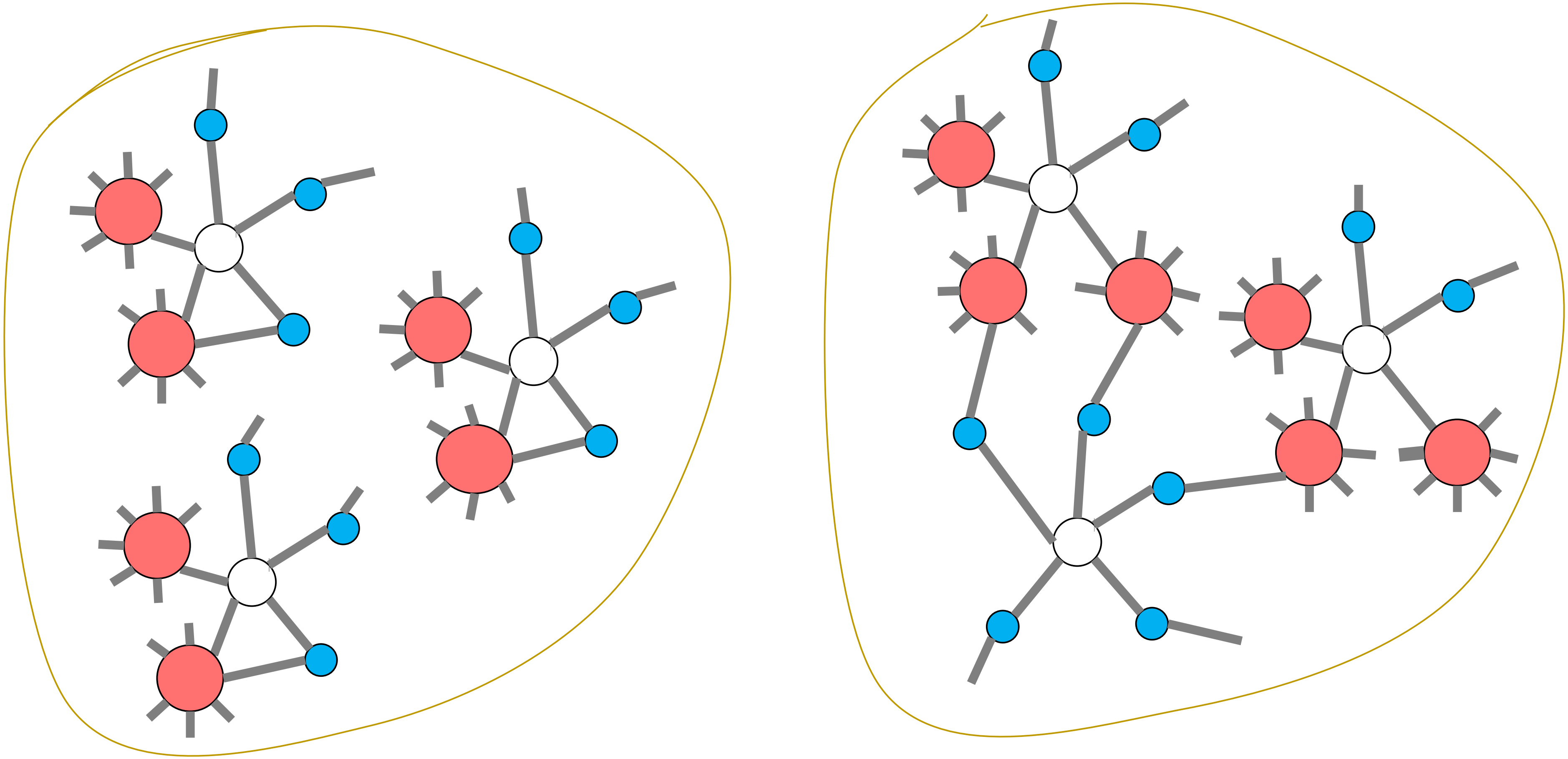
Degree correlations among node's neighbors in real-world networks are often large



Third-order structure enhances paradoxes

Neighbors' degrees are not correlated*

Neighbors' degrees are correlated*



***same 1K and 2K structure**

Third-order structure enhances paradoxes

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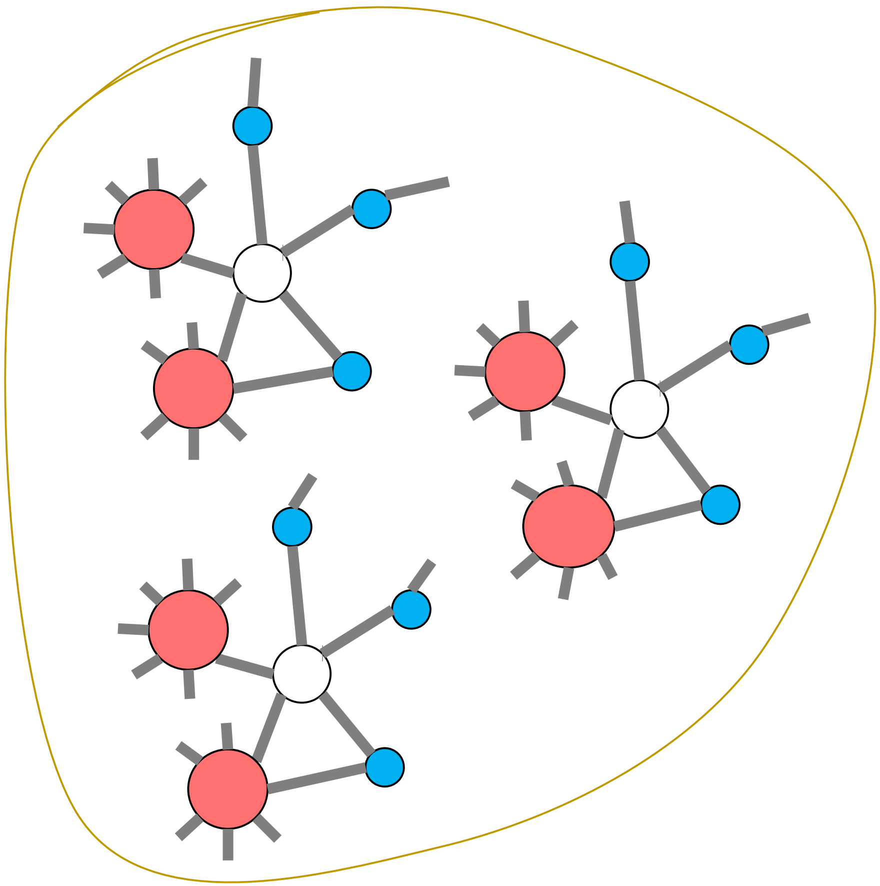
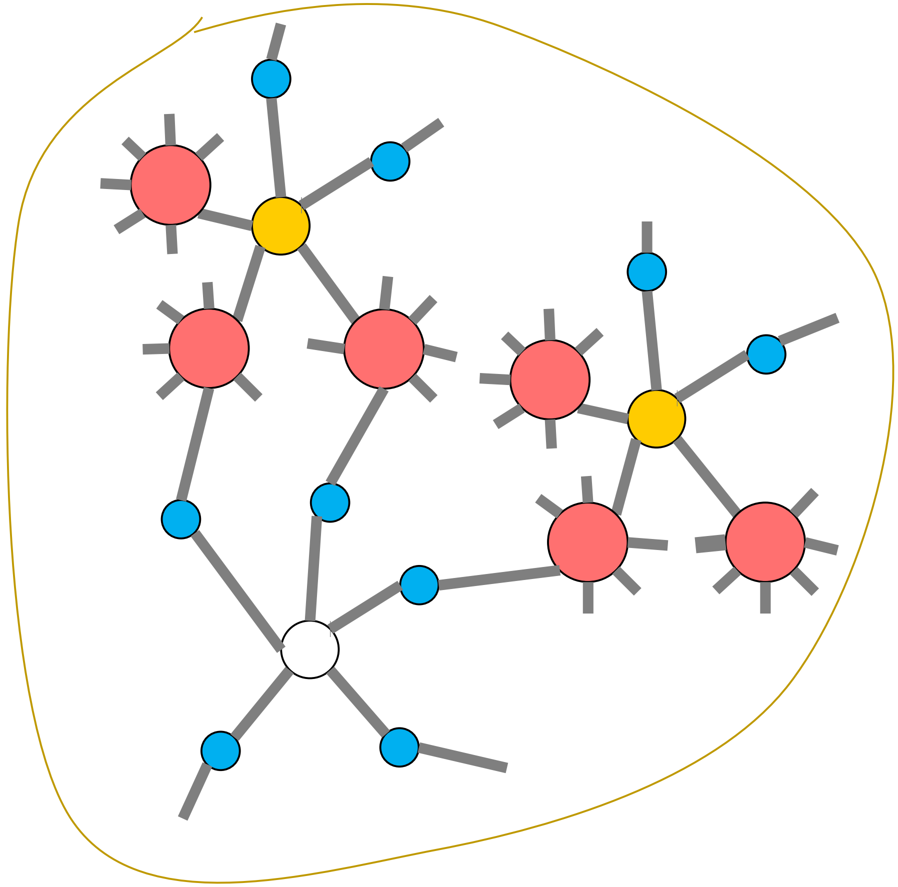
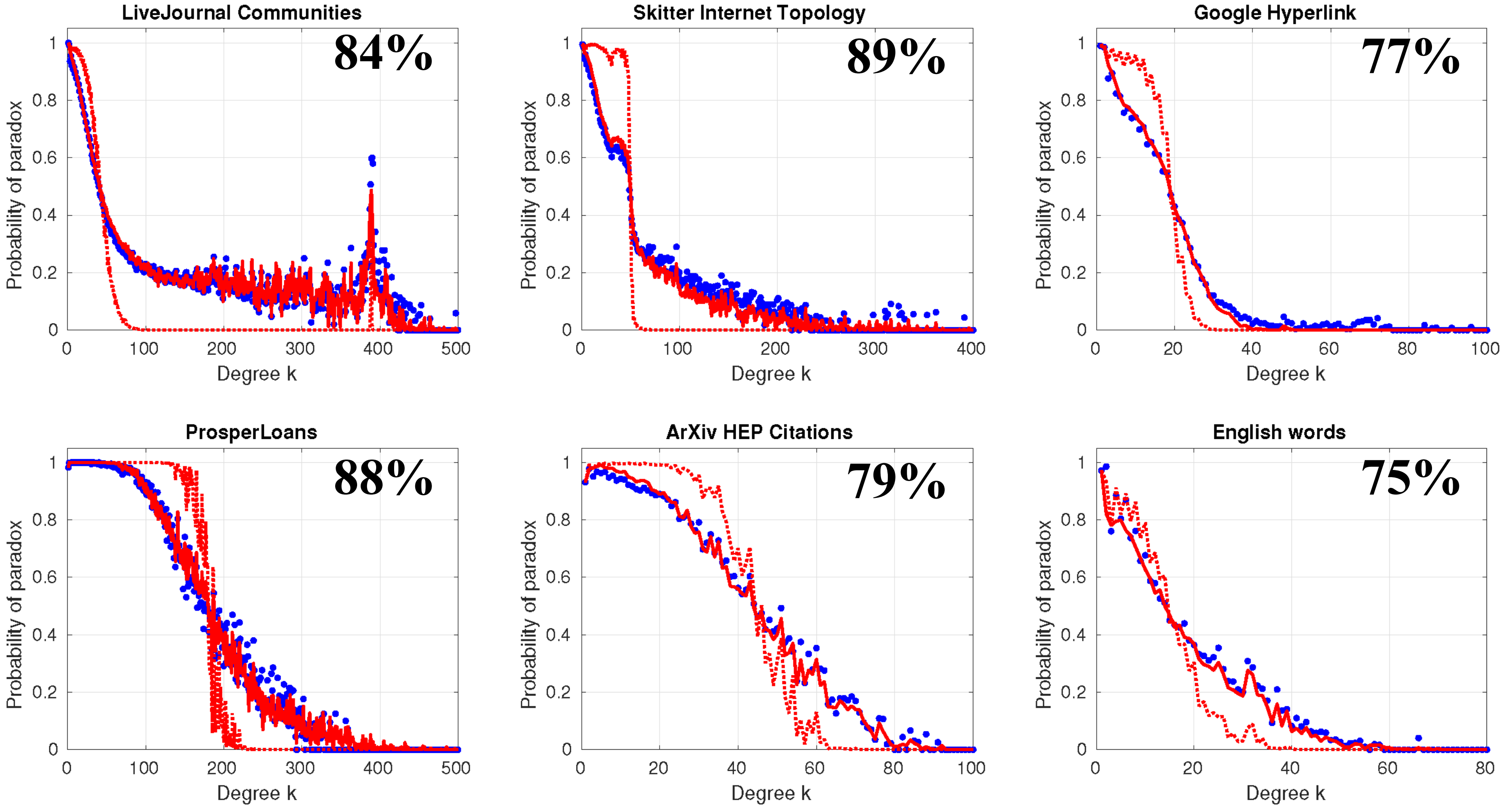


Neighbors' degrees are correlated*



***same 1K and 2K structure**

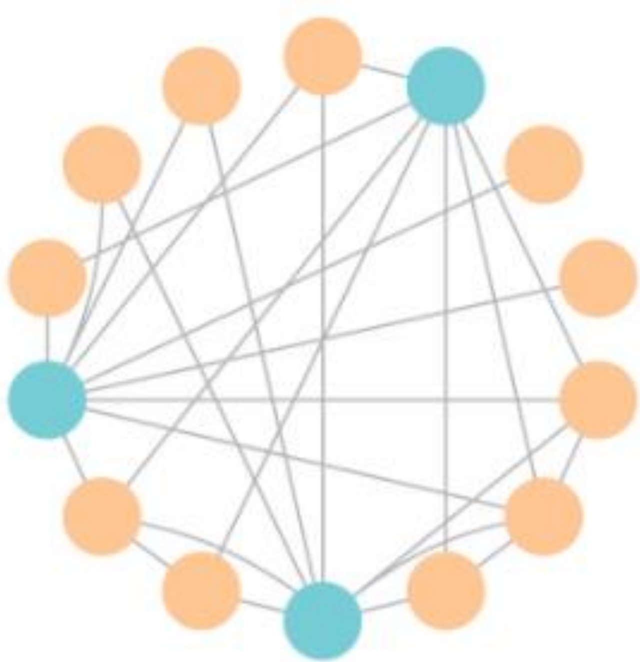
Fraction of degree-*k* nodes experiencing the paradox in real-world networks ● ; predictions of the 2*K* model ··· and the 3*K* model —.



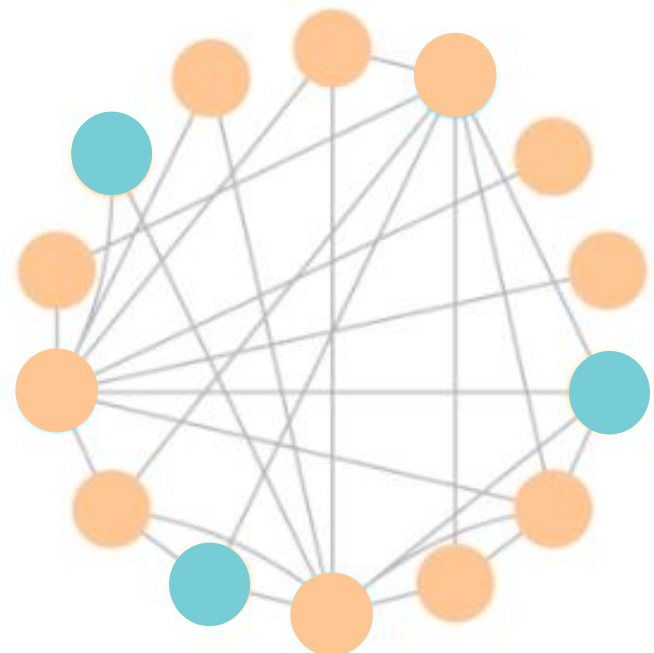
[Wu et al (2017) "Neighbor-neighbor correlations explain measurement bias in networks" *Scientific Reports* 7]

From friendship paradox to "majority Illusion"

Nodes have a binary trait: active/not, yellow/blue, heavy drinker/teetotaler, …



Many think that blue is common



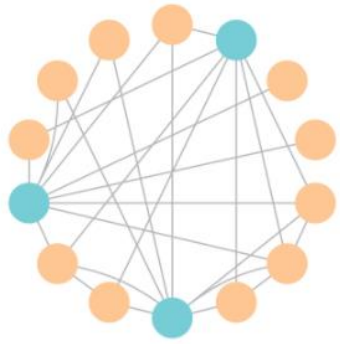
Blue does not appear common

[Lerman, Wu & Yan (2016) The "Majority Illusion" in Social Networks, in *Plos One*.]

Network structure amplifies majority illusion

More nodes will think that blue is very common when:

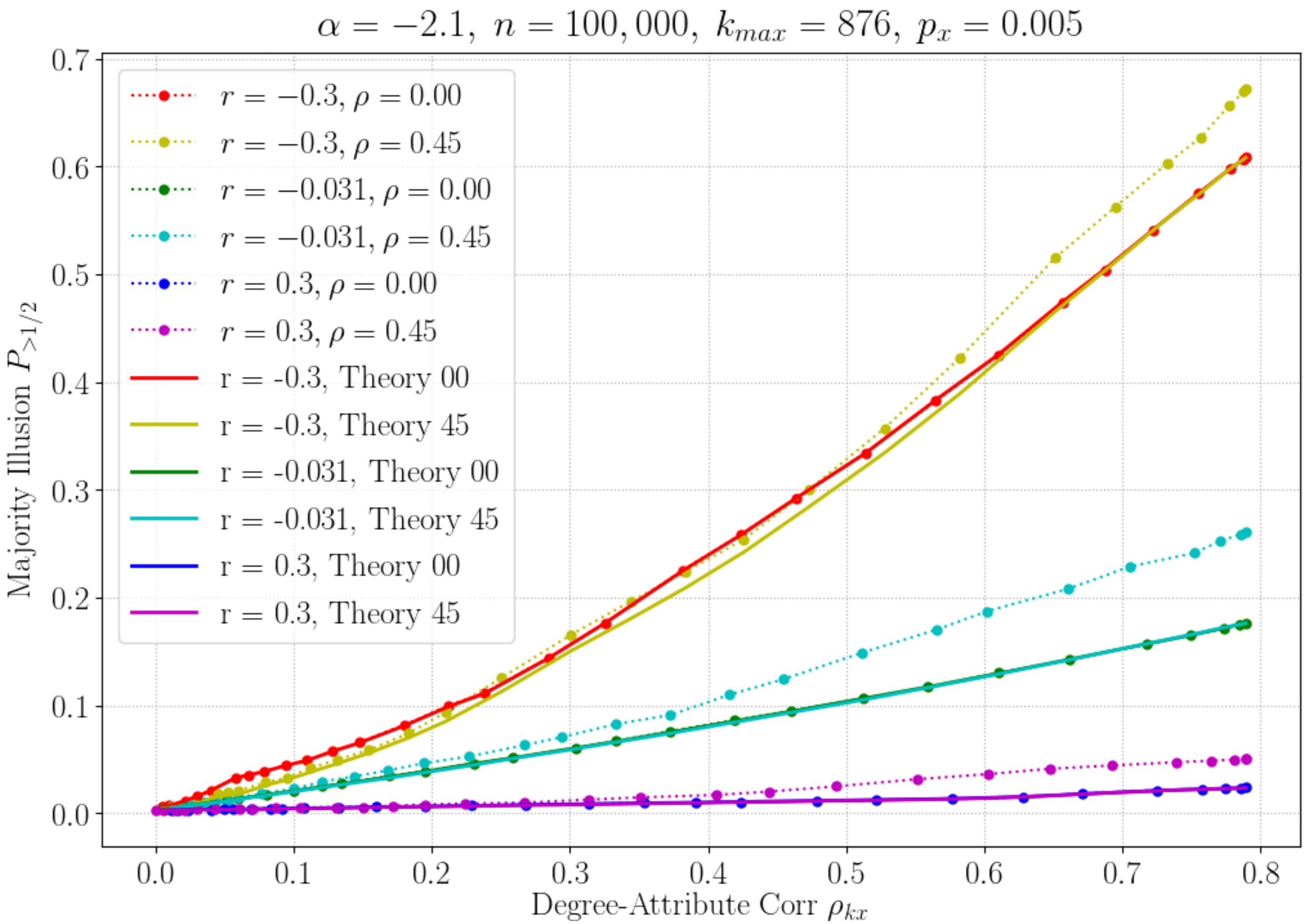
- Higher degree nodes are more likely to be blue: degree-trait (k-x) correlation
- High degree nodes link to low degree nodes: degree disassorativity ($r_{2k}<0$)
- Neighbors tend to have similar degree: neighbor assortativity ($r_{3k}>0$)



[Lerman, Wu & Yan (2016) The "Majority Illusion" in Social Networks, in *Plos One.*]

Network structure amplifies majority illusion

Fraction of nodes experiencing the majority illusion in a synthetic network with 0.5% active nodes;



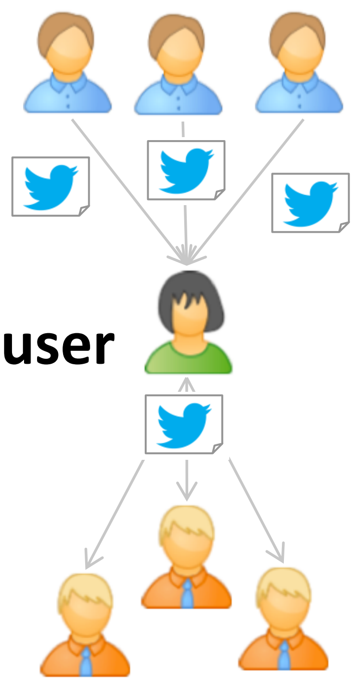
[Lerman, Wu & Yan (2016) The "Majority Illusion" in Social Networks, in *Plos One.*]

Friendship paradox in directed networks

friends

in-degree d_i

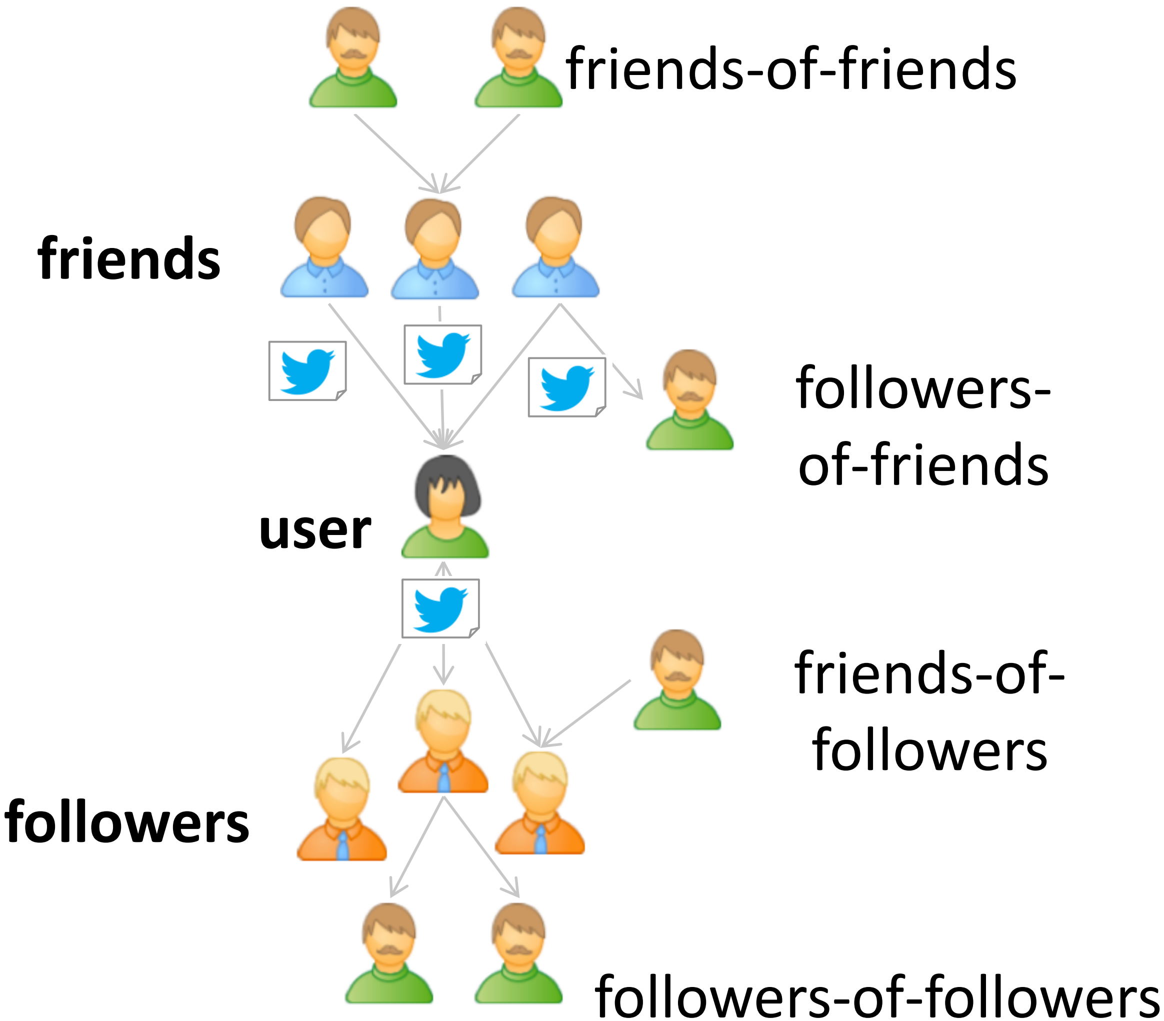
user



followers

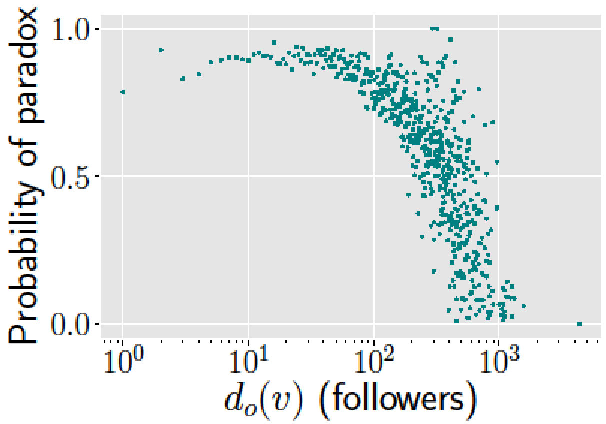
out-degree d_o

Friendship paradox in directed networks

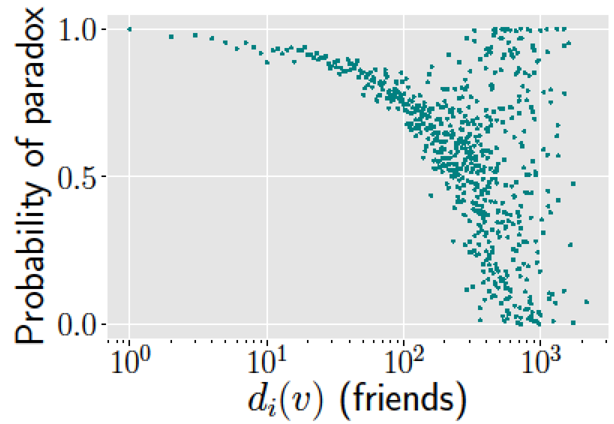


Probability a node experiences a paradox

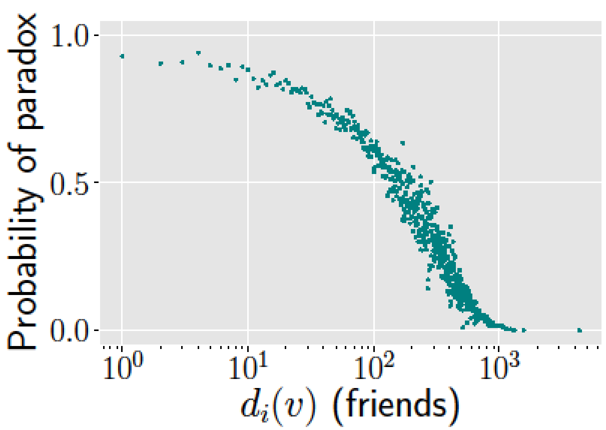
friends have more followers



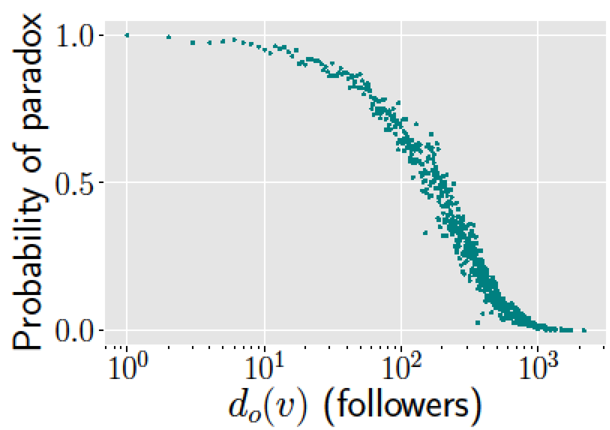
followers have more friends



friends have more friends



followers have more followers



Local perception bias

- Popularity of a (binary) attribute: probability a random node *v* has value *f*(*v*)

$$\mathbf{E}\{f(v)\}$$

- Local perception of node *v* about popularity of an attribute *f* is the fraction of her friends with attribute

$$q(v) = \frac{\sum_{u \in friends(v)} f(u)}{d_i(v)}$$

- Local perception bias: nodes perceive the attribute *f* to more popular than it actually is

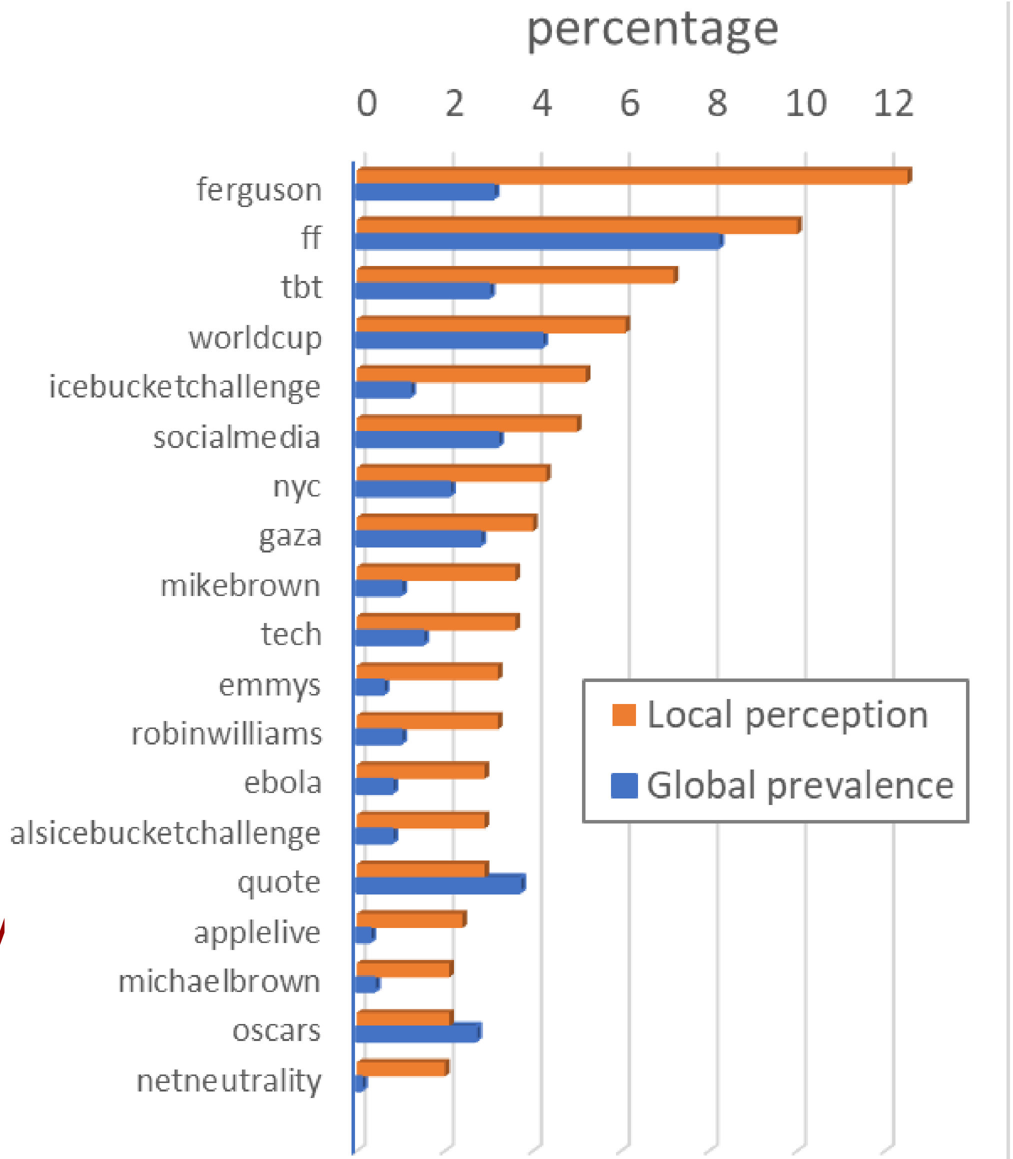
$$\mathbf{E}\{q(v)\} \geq \mathbf{E}\{f(v)\}$$

Global popularity vs local perception on Twitter

Twitter data

- Time period
  - Summer 2014
- Network
  - 5K users + tweets
  - Their 600K friends + tweets
- Hashtags
  - 18M hashtags
  - Focus on 1K most popular hashtags, used by >1K people

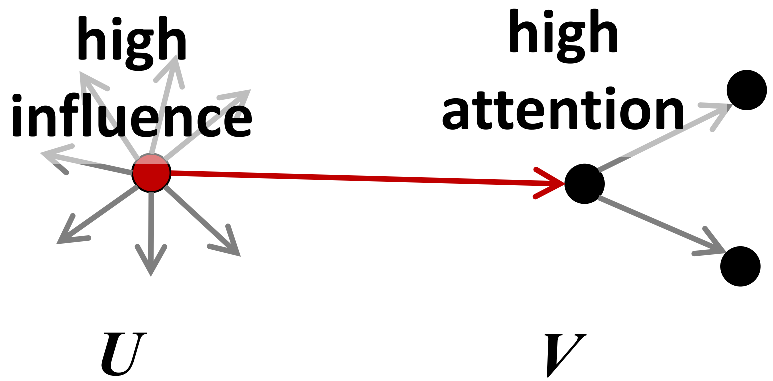
Compare perceived popularity of hashtags to their actual popularity



Conditions for local perception bias

Local bias exists if:

- Higher out-degree (high influence) tend to have the attribute.
- Lower in-degree nodes (high attention) tend to follow nodes with attribute.



Theorem: $(\mathrm{E}\{q(v)\} \geq \mathrm{E}\{f(v)\})$ **if**

$$\mathrm{Cov}\{f(X), d_o(X)\} \geq 0 \quad \textit{and,}$$

$$\mathrm{Cov}\{f(U), \mathcal{A}(V) | (U,V) \sim \mathrm{Uniform}(E)\} \geq 0.$$

Polling

What is the right question to ask in a poll?

Estimate the true prevalence of an attribute through polls

- Estimate the fraction of liberals vs conservatives, heavy drinkers vs teetotalers, people who used a hashtag vs not, …
- …with limited budged b

Polling:

1. Intent Polling (IP): [b random nodes] Will you vote for X?
2. Node Perception Polling (NPP): [b random nodes] What fraction of your friends will vote for X?
3. **Follower Perception Polling (FPP): [b random followers] What fraction of your friends will vote for X?**
   - aggregates perceptions of more people

Bias & variance of FPP

- Bias of the polling estimate (error) T

$$Bias(T) = \mathrm{E}\{T\} - \mathrm{E}\{f(X)\} = \frac{\mathrm{Cov}(f(X), d_o(X))}{\hat{d}}$$

- Variance is bounded by λ_2, second largest eigenvalue of the symmetrized adjacency matrix of the network

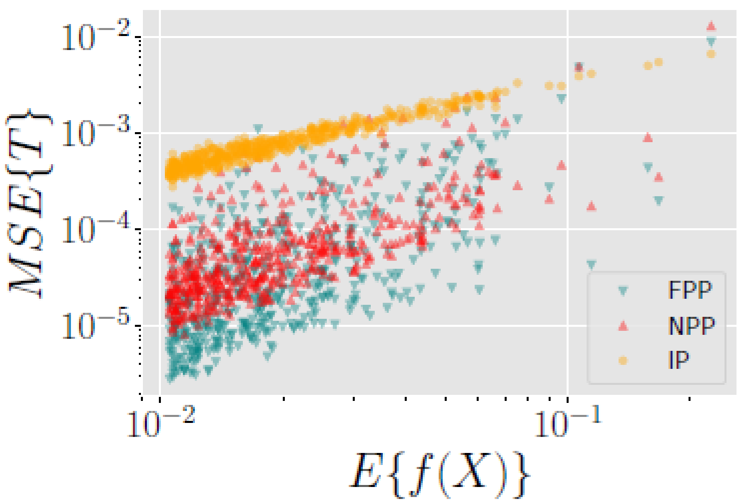
- Mean squared error of the polling estimate T:

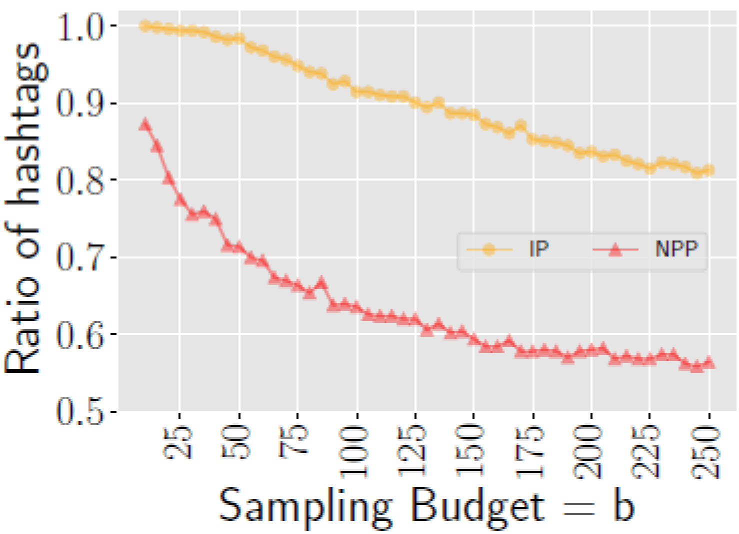
$$\mathrm{MSE}\{T\} = \mathrm{E}\left\{\left[T - \mathrm{E}\left(f(X)\right)\right]^2\right\} = \mathrm{Bias}\{T\}^2 + \mathrm{Var}\{T\}$$

FPP polling algorithm is more efficient

When used to estimate the true popularity of Twitter hashtags, FPP has lower variance and MSE.

For a given budget, i.e., number of nodes sampled, it outperforms other polling methods on many hashtags.





To summarize

- Network structure can systematically bias local perceptions
  - Making a rare attribute appear far more common than it is, under some conditions
- Open questions: What is the impact of network bias on
  - Collective dynamics in networks, e.g., contagious outbreaks
  - Network control and intervention
  - Psychological well-being
    - Your friends are happier that you are (Bollen et al. 2016)
    - Your co-authors are more prestigious than you are (Eom & Jo 2014)
    - Social comparison theory

THANK YOU!

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Questions?

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